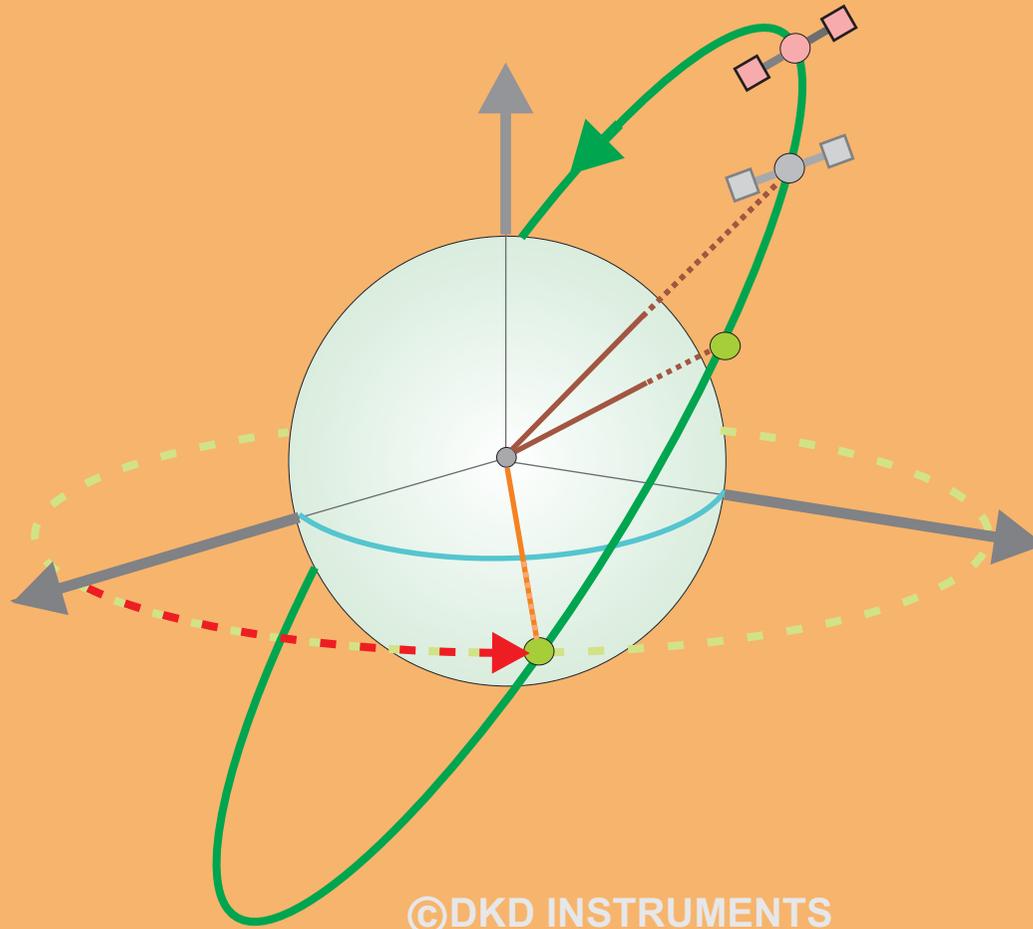


FUNDAMENTALS OF GPS RECEIVERS
A HARDWARE APPROACH
by
DAN DOBERSTEIN



GPS FUNDAMENTALS A HARDWARE APPROACH BOOK I of II

**BY DAN DOBERSTEIN
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BOOK I

Introduction to GPS FUNDAMENTALS

This text is essentially a primer for BOOK II, “GPS Fundamentals, A Hardware Approach”. The motivation for this portion of the two-book set was dissatisfaction with the introductory material in BOOK II, which was written first. A choice was made to leave the original material intact (warts and all) and re-write the introductory material from scratch. As it progressed it got larger, to the point it was realized it was a separate text.

The goal of the introductory material was to get to the core concepts as soon as possible. Some of the top concepts were;

- How to measure Distance using Clocks
- Resolution of Range Measurements
- Pseudo Range
- The GPS Clock
- Difference and meaning between Code Phase & Carrier Phase Measurements
- Observed Clock Phase is Range LOS
- Observed Clock Rate is Doppler or Velocity LOS
- Observed Clock Rate-Rate is Acceleration LOS
- Cycle Slips

These concepts are not only fundamental but they are also least likely to be defined *before* being used in too many places. My own experience in industry and conversation with other engineers was the fundamentals were often not understood. Range resolution is an excellent example of this. Once you know the model of the GPS clock you should be able to approximate range resolution of code or carrier phase measurements quickly. I was surprised at how many people in GPS did not know this information in terms of deriving it from basic system information.

In order to tackle the above goals I decided to start from the basics. In GPS, that means a complete understanding of how clocks work. To help readers understand the basics of clocks (and GPS) I *purposefully* chose *rotary mechanical* clock models over straight mathematics or linear clock models. I have found the rotary clock mechanical models extremely useful in understanding what exactly is going on inside GPS receivers. I maintain we all know how rotary mechanical clocks work. The math behind it should come *after* that innate understanding is brought to the surface. Lastly rotary time scales more accurately reflect what is happening in GPS receiver than linear time scales. Most every signal of interest and the operations on them follow the math of rotary machines..

Throughout this primer text hardly a page will go by without a picture or figure of a rotary mechanical clock. That was done for a reason. For even if you skim read or get stuck all those clock images will start to leave their mark; In GPS, Clocks are King. It is my opinion that too much of what has been written about GPS forgets to *explicitly* remind the reader that nearly everything in GPS is about the clock(s), directly or indirectly. I have seen texts that go on for pages and pages forgetting to tell the reader they are really discussing clocks even if they don't say so explicitly.

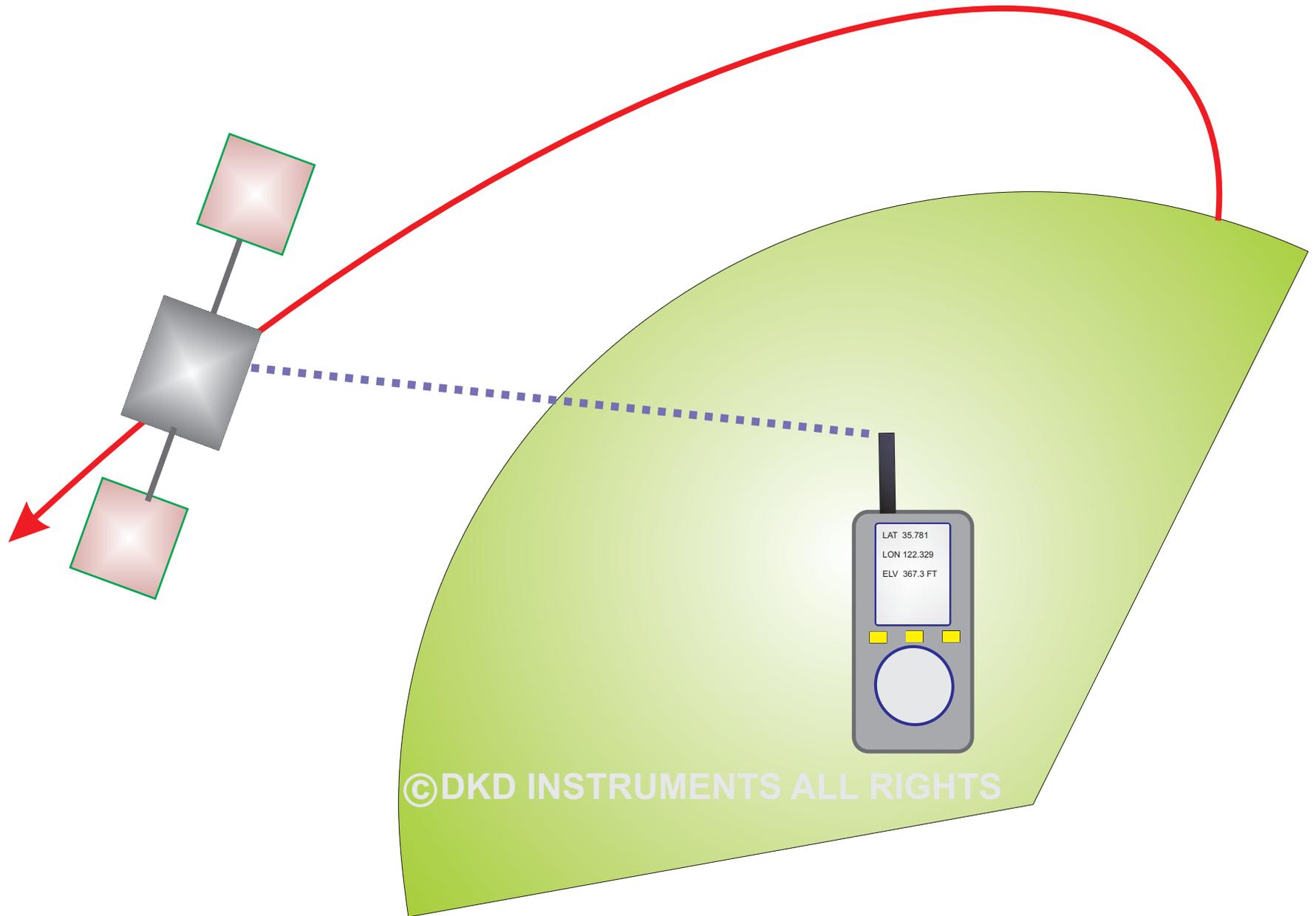
Needless to say this text is not for everyone. In addition the style of BOOK I is a cross between a presentation format and a traditional technical book. The ideal goal was a text that explained the basics with images, not equations. But without true animation that's a tough thing to do.

PART I
BASIC RANGE MEASUREMENTS
USING A MECHANICAL ROTARY CLOCK MODEL



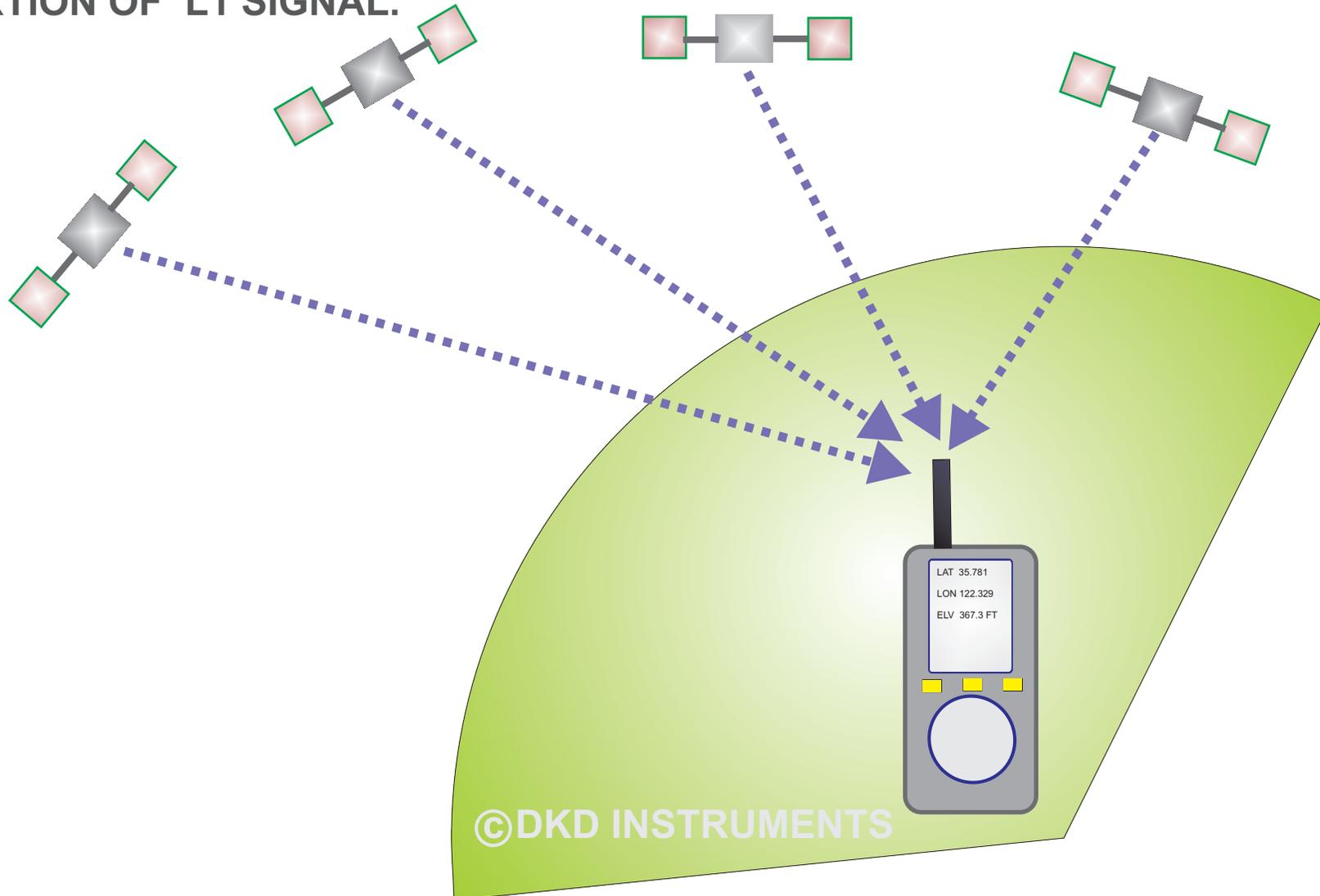
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GPS RECEIVERS USE SATELLITES TO MEASURE & COMPUTE THEIR POSITION ON THE EARTH

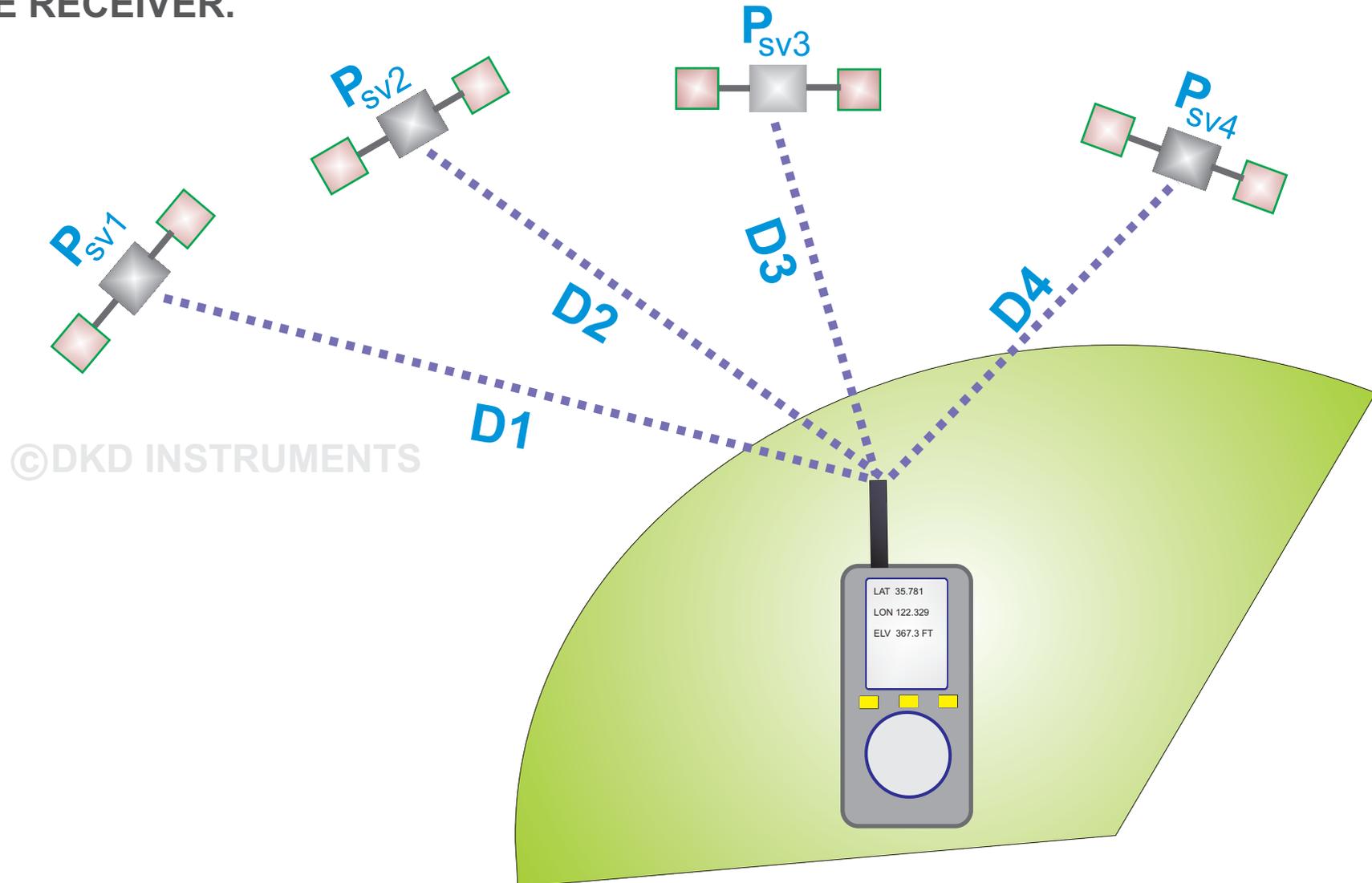


TO DETERMINE ITS POSITION A GPS RECEIVER TYPICALLY NEEDS TO RECEIVE DATA FROM AT LEAST 4 SATELLITES. GPS RECEIVERS DO NOT TRANSMIT ANY INFORMATION TO THE SATELLITES

THE INFORMATION IS SENT USING RADIO WAVES. THERE ARE THREE BANDS OR SIGNALS SENT , L1 , L2 AND L5. THIS TEXT IS MOSTLY ABOUT THE CIVILIAN PORTION OF L1 SIGNAL.

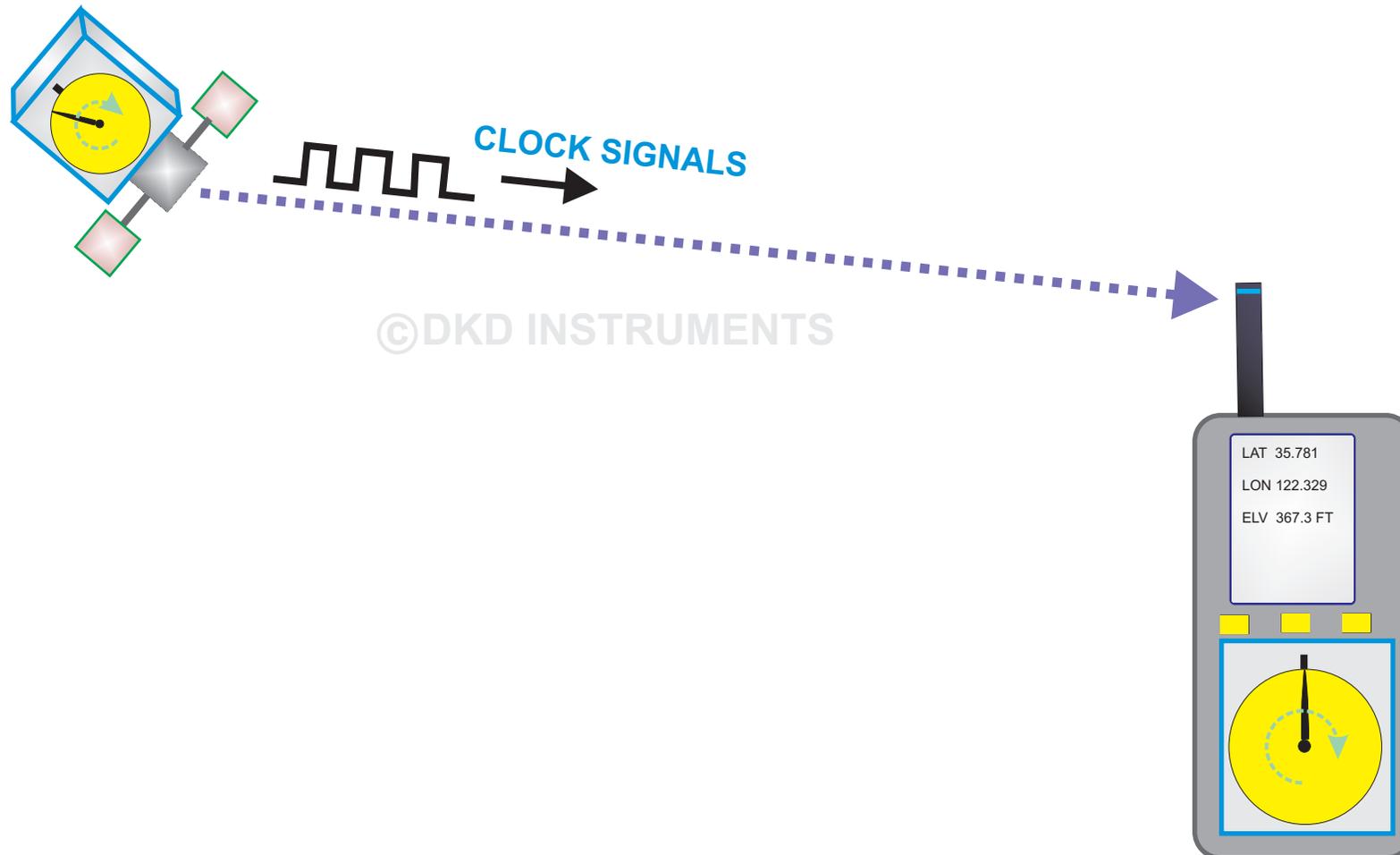


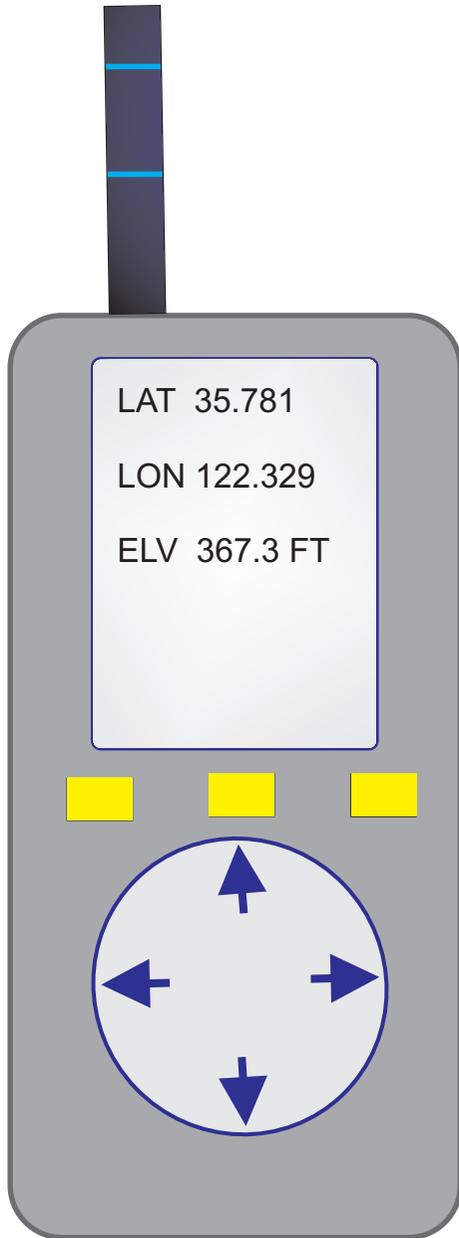
THE RECEIVER NEEDS TO KNOW THE POSITIONS OF THE FOUR SATELLITES AND THE DISTANCES TO THEM TO COMPUTE ITS POSITION ON THE EARTH AND THE ERRORS (RATE AND PHASE) ON ITS OWN INTERNAL CLOCK. THE POSITION OF THE SATELLITES IS COMPUTED FROM ORBITAL INFORMATION RECEIVED FROM THE SATELLITES OR INTERNET . THE DISTANCES TO THE SATELLITES (CALLED RANGE) IS MEASURED BY THE RECEIVER.



THE DISTANCE OR *RANGE* TO EACH SATELLITE IS MEASURED USING CLOCKS. THE SATELLITES EACH HAVE A CLOCK AND THEY SEND SIGNALS DERIVED FROM THAT CLOCK DOWN TO THE RECEIVER. THE RECEIVER COMPARES THESE RECEIVED CLOCK SIGNALS TO ITS OWN CLOCK AND COMPUTES THE RANGE BY USING THE TIME DIFFERENCE MULTIPLIED BY THE SPEED OF LIGHT.

THE PROCESS OF RECEIVING AND COMPARING A SATELLITE CLOCK IS PART OF WHAT IS CALLED TRACKING A SATELLITE.



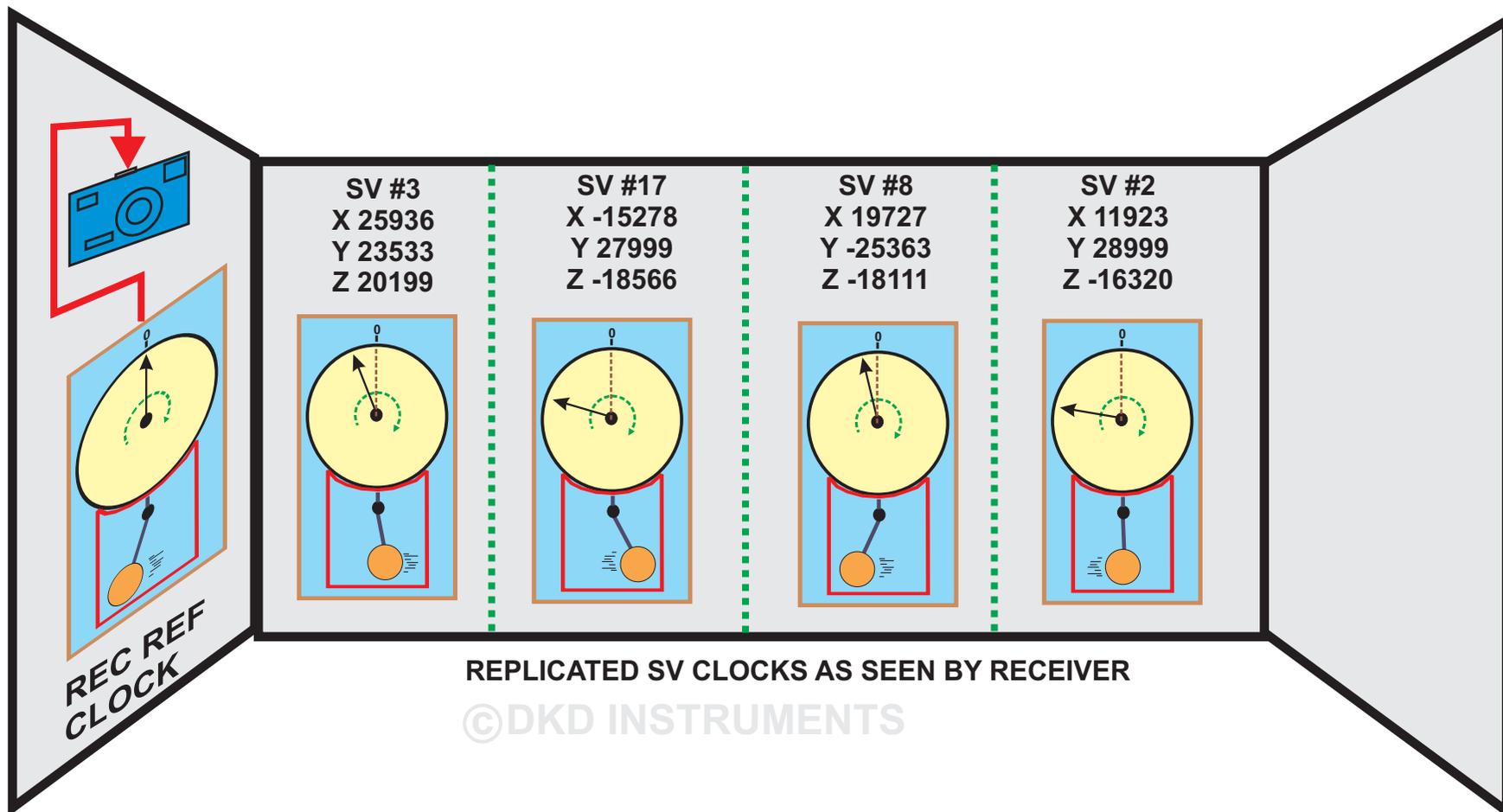


**NOW THAT WE KNOW A BIT
ABOUT GPS RECEIVERS WHAT IF
WE COULD LOOK INSIDE A GPS
RECEIVER?**

WHAT WOULD WE SEE?

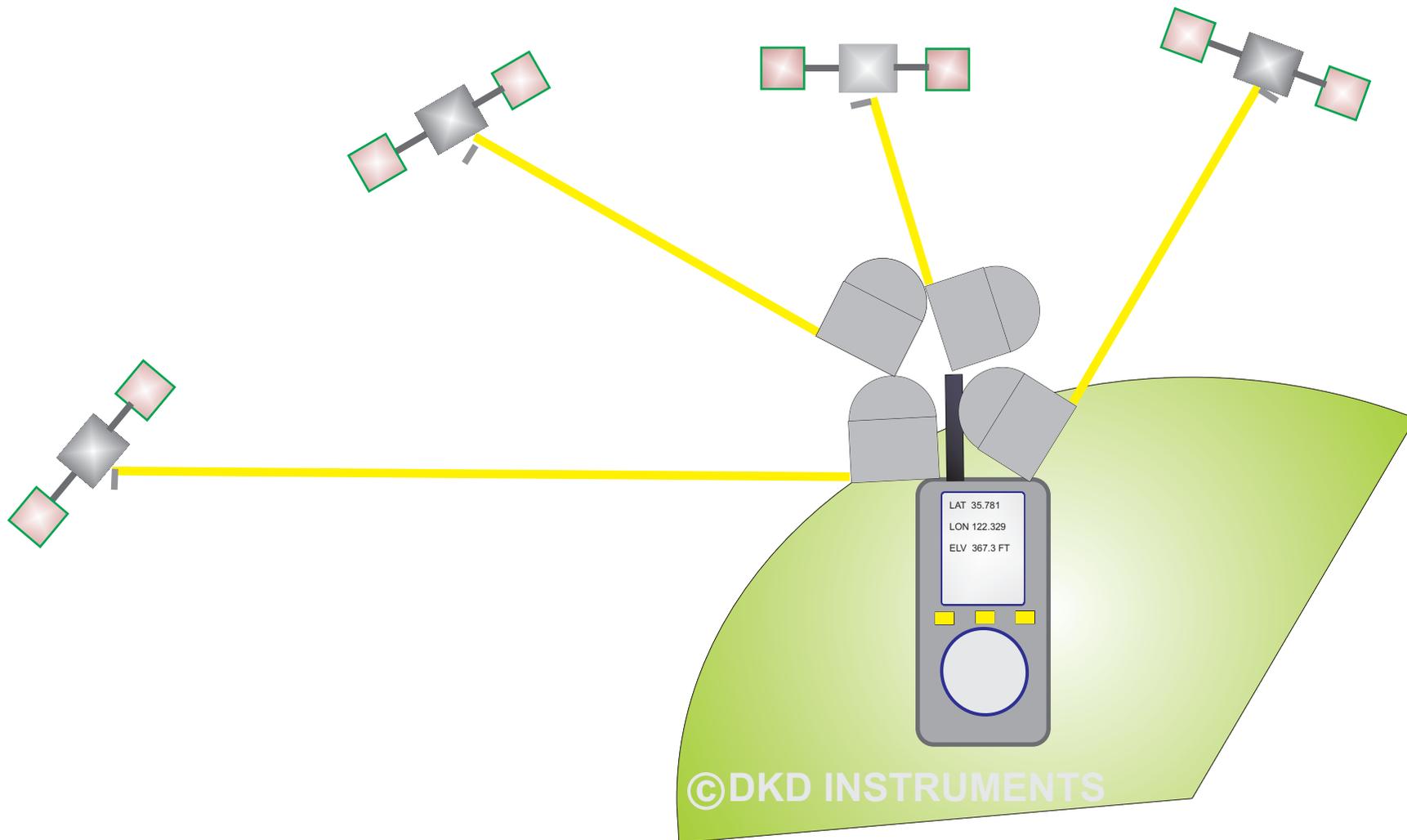
WHAT WE WOULD SEE IS A ROOM FULL OF CLOCKS! EACH OF THE FOUR SATELLITES CLOCKS IS THERE ALONG WITH THE RECEIVERS REFERENCE CLOCK. IN ADDITION THE POSITIONS AS CALCULATED FROM THE ORBITAL INFORMATION OF EACH SATELLITE ARE DISPLAYED ABOVE ITS CLOCK .

THE CAMERA TAKES PICTURES OF ALL THE CLOCKS AND POSITION INFORMATION WHICH USED TO COMPUTE POSITION OF THE RECEIVER.

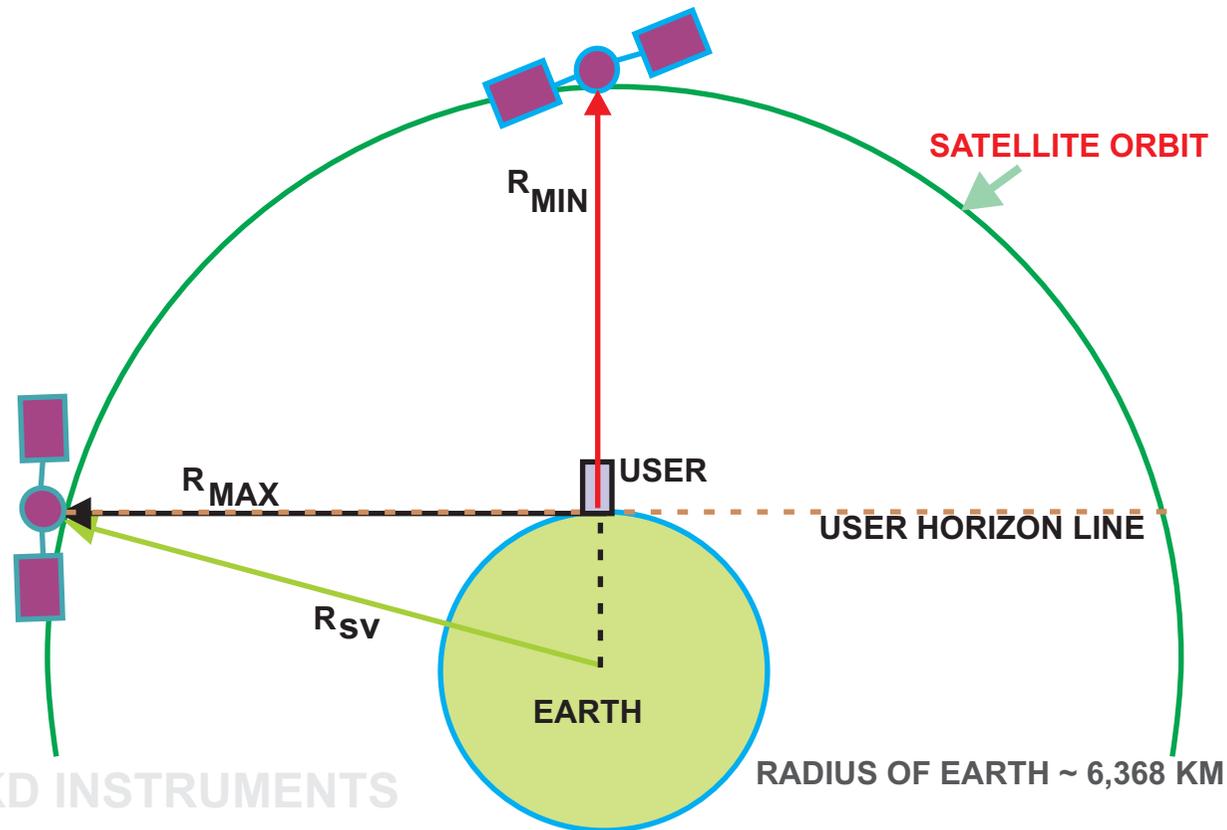


AS EACH SATELLITE CLOCK IS COMPARED TO THE RECEIVERS CLOCK. THE RANGE TO THAT SATELLITE IS CONTINUOUSLY MEASURED BY THE RECEIVER. THIS ALLOWS THE RECEIVER TO KEEP TRACK OF THE CHANGING RANGE TO EACH SATELLITE DUE TO THE ORBITAL MOVEMENT OF THE SATELLITE.

WE CAN MODEL THE MEASUREMENT OF RANGE TO EACH SATELLITE AS A VERY LONG TAPE MEASURE STRETCHING FROM THE RECEIVER TO THE SATELLITE. THE RANGE TO EACH SATELLITE IS NOW SHOWN ON EACH TAPE MEASURE. AS THE RANGE CHANGES TO EACH SATELLITE THE TAPE MEASURE GOES IN AND OUT.



THE RANGE FROM THE RECEIVER TO A GPS SATELLITE & PHYSICAL CONSTANTS (FOR A GPS SATELLITE ORBIT THAT PASSES DIRECTLY OVERHEAD)



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USER TO SATELLITE DATA

$R_{MIN} \sim 20,000\text{KM}$

$R_{MAX} \sim 25,593\text{ KM}$

Min Time Delay $\sim 66\text{msec}$
Max Time Delay $\sim 86\text{msec}$

GPS SATELLITE DATA

MEAN ALTITUDE ABOVE SEA LEVEL $\sim 20,000\text{KM}$

R_{sv} = RADIUS OF ORBIT WRT CENTER OF THE EARTH $\sim 26,550\text{ KM}$

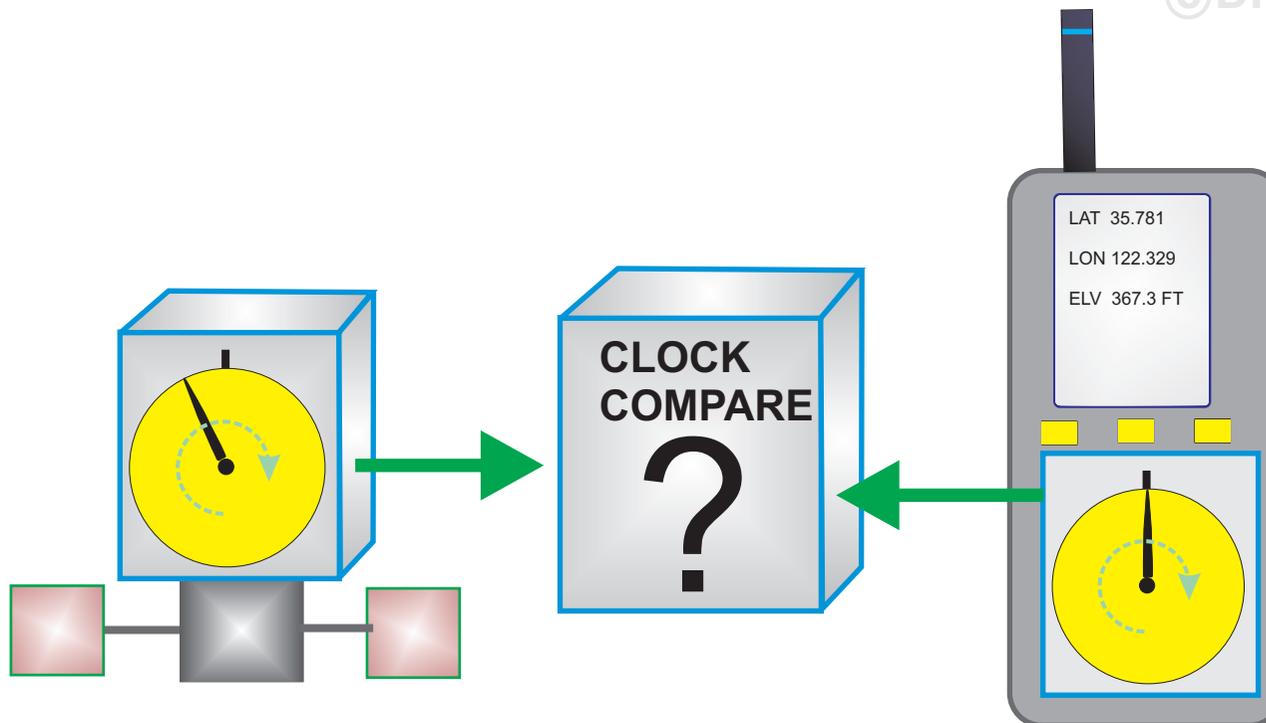
ORBITAL RATE : 2 ORBITS IN 24 HOURS

ORBITAL SPEED $\sim 3874\text{M/SEC}$

WE NOW KNOW THAT GPS RECEIVERS MUST MEASURE THE RANGE TO EACH SATELLITE. WE ALSO KNOW THAT THIS DONE BY A CLOCK COMPARISON PROCESS.

BUT HOW DOES THAT WORK? IN ORDER TO ANSWER THAT QUESTION REQUIRES A COMPLETE UNDERSTANDING OF CLOCKS , HOW TO USE THEM TO MEASURE DISTANCE, CLOCK TERMINOLOGY AND FOR THIS TEXT HOW MECHANICAL ROTARY CLOCKS FUNCTION.

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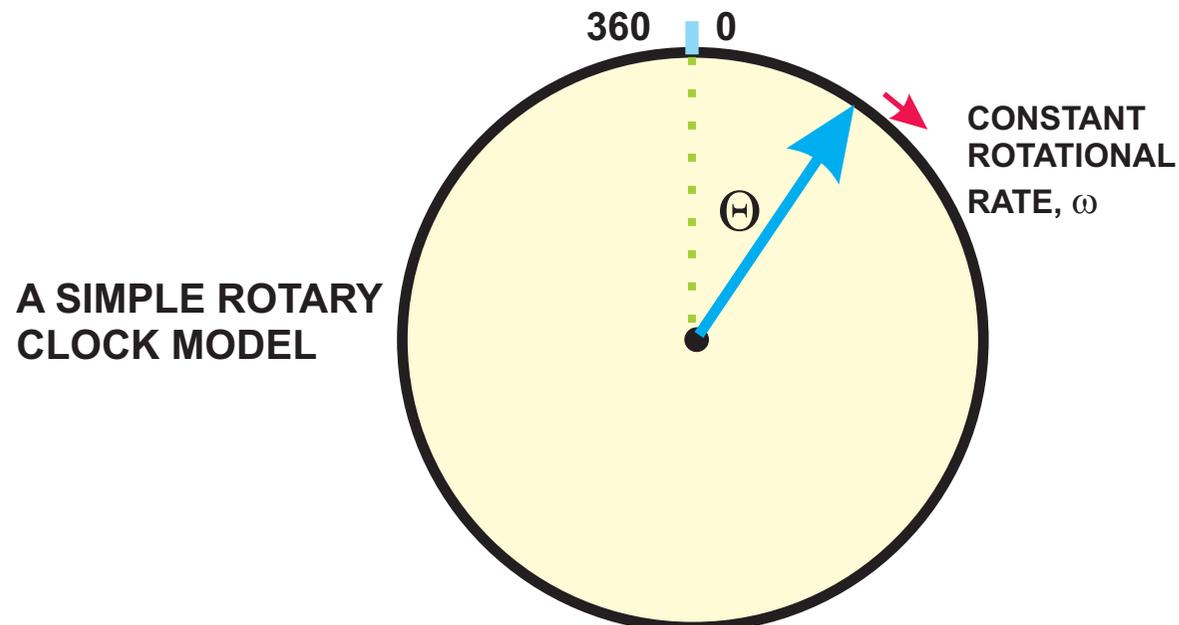


IN ORDER TO UNDERSTAND HOW RANGE IS MEASURED USING CLOCKS THE READER MUST BE WELL VERSED IN HOW CLOCKS WORK, THE TERMINOLOGY OF CLOCKS AND OBSERVATIONS OF CLOCKS THAT ARE MOVING.

IF THE READER IS NOT FAMILIAR WITH CLOCK BASICS THEY SHOULD READ THE ***BASICS OF CLOCKS*** APPENDIX BEFORE PROCEEDING.

IN ADDITION WE WILL BE USING ROTARY TYPE, MECHANICAL CLOCK MODELS EXTENSIVELY IN THIS TEXT. FOR THOSE UNFAMILIAR WITH SUCH CLOCKS PLEASE SEE APPENDIX II ***MECHANICAL MULTI-DIAL ROTARY CLOCKS AND A MULTI-DIAL MODEL GPS CLOCK***

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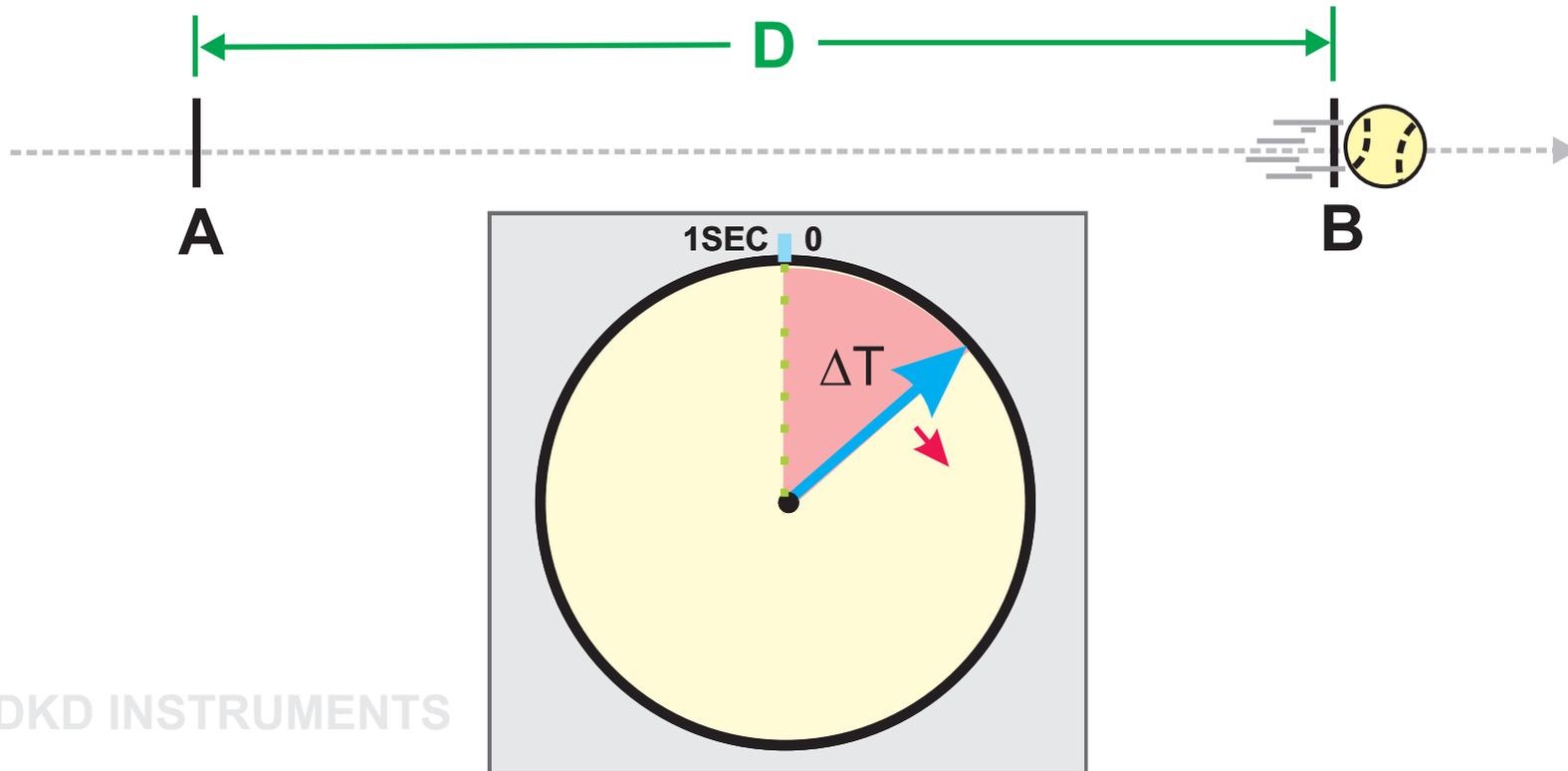


MEASURING DISTANCE USING A CLOCK AND A CONSTANT SPEED BASEBALL
BELOW WE SEE A BASEBALL TRAVELING AT A CONSTANT SPEED, V , IT PASSES FROM POINT A TO POINT B

THE ANALOG CLOCK SHOWS THE TIME IT TOOK TO GO FROM POINT A TO POINT B AS ΔT

WE HAVE PURPOSELY HAD THE BASEBALL PASS POINT A AT TIME ZERO ON OUR CLOCK.

WE CAN COMPUTE THE DISTANCE, D , BETWEEN POINT A AND B BY $D = V * \Delta T$

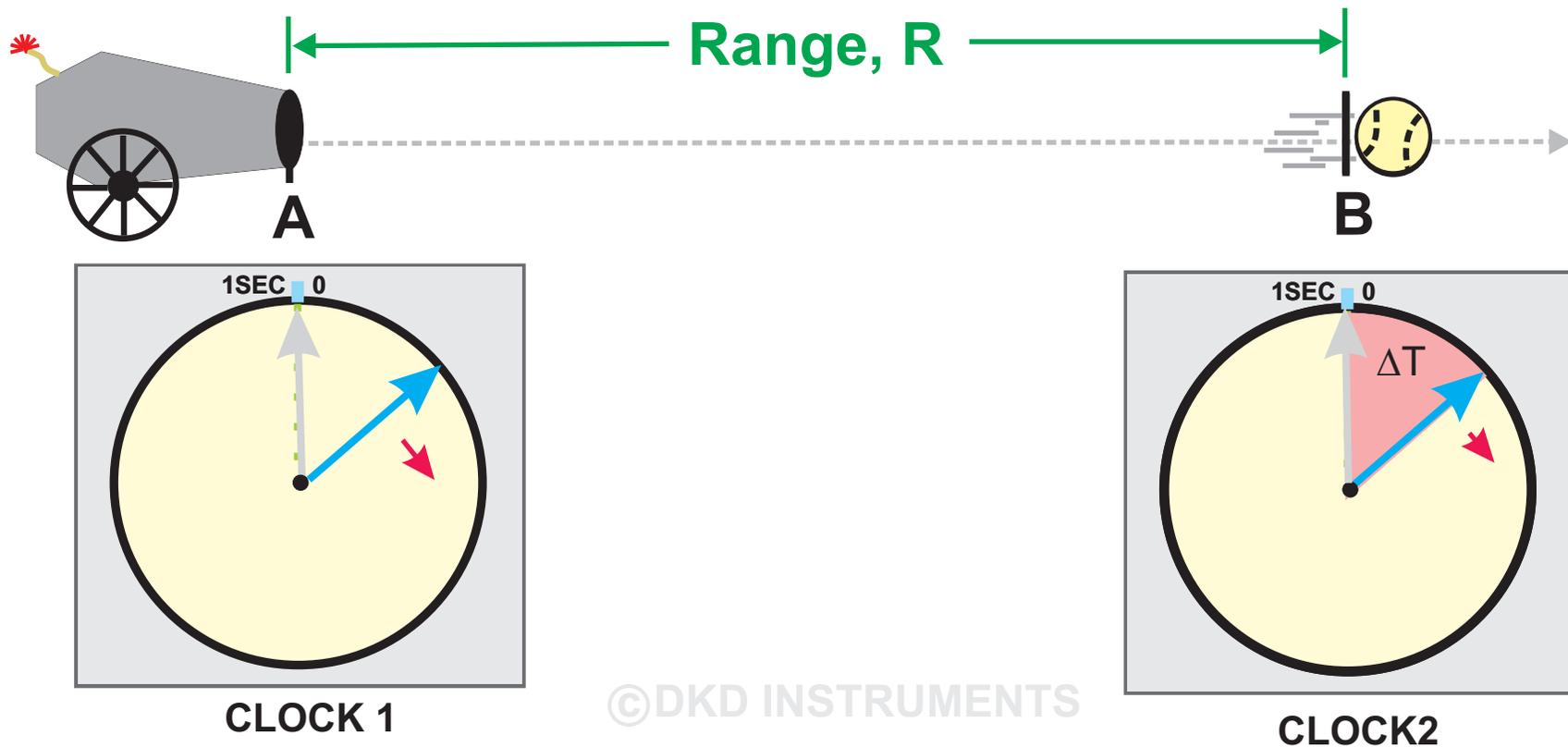


TWO CLOCKS AND A BASEBALL SHOT FROM CANON

WHEN CLOCK1 HITS ZERO IT FIRES A BASEBALL AT POINT B WITH CONSTANT SPEED V . THIS MOMENT IS DEPICTED ON THE ANALOG CLOCKS WITH THE GREY POINTER.

WHEN THE BASEBALL PASSES THE CLOCK2 THE TIME IS RECORDED, ΔT . WE CAN COMPUTE THE RANGE R , BETWEEN POINT A AND B BY $R = V \cdot \Delta T$

THIS ONLY WORKS IF CLOCK1 AND CLOCK2 REMAIN AT IDENTICAL RATES AND HAVE ZERO PHASE ERROR BETWEEN THEM.

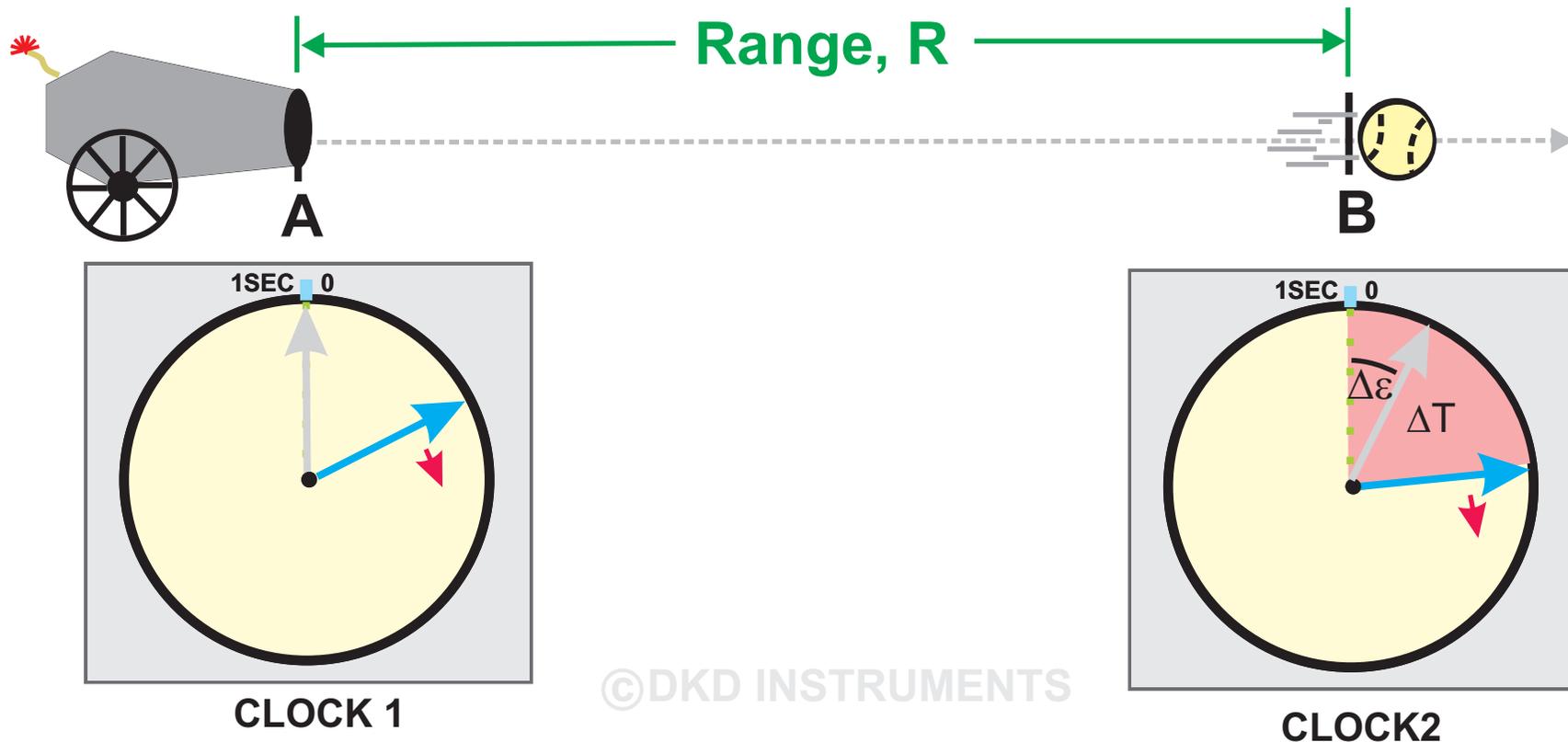


TWO CLOCKS, A BASEBALL, AN ERROR ON CLOCK2 => PSEUDO RANGE

THE OBSERVER AT CLOCK2 RECORDS THE TIME WHEN THE BASEBALL PASSES AS ΔT . THE OBSERVER IS UNAWARE HIS CLOCK HAS A PHASE ERROR wrt CLOCK1 OF $\Delta \epsilon$, WHICH ALSO MEANS ΔT IS IN ERROR.

WITH $\Delta \epsilon$ AS SHOWN ΔT IS LARGER THAN THE CORRECT VALUE, THEREFORE THE RANGE WILL BE INCORRECTLY CALCULATED AS LONGER BY $\Delta \epsilon * V$ [NOTE THAT THE CORRECTED TIME OF FLIGHT IS $\{ \Delta T - \Delta \epsilon \}$]

WHEN A CALCULATED RANGE HAS AN ERROR DUE TO A CLOCK PHASE ERROR IT IS CALLED A **PSEUDO RANGE**.



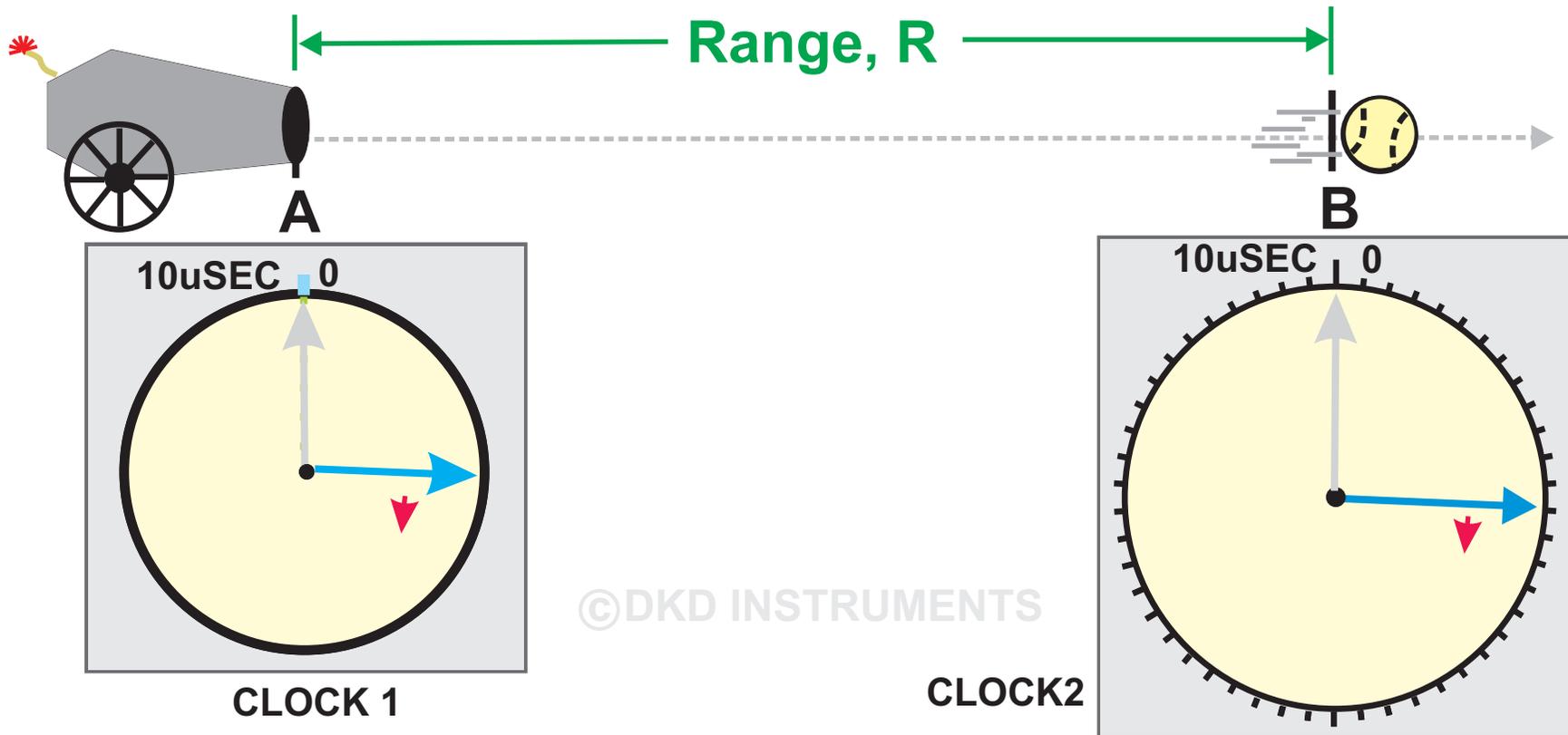
MEASURE RANGE WITH TWO CLOCKS AND A SPEED OF LIGHT BASEBALL

A BASEBALL IS NOW SHOT FROM A HYPER CANNON AT TIME ZERO WITH SPEED OF LIGHT , C, EVERY 10uSEC. AS BEFORE OUR TWO CLOCKS ARE SYNC'ED.

THE ANALOG CLOCK2 IS REPLACED BY A DIGITAL CLOCK WITH 50 TIC'S IN ONE CYCLE , WITH ONE CYCLE OCCURRING EVERY 10uSEC.

NOW OUR RECORDED TIME AT POINT B IS QUANTIZED TO A RESOLUTION OF $10\mu\text{SEC}/50 = 200 \text{ NANO-SEC}$.

OUR MEASURED RANGE IS NOW ALSO QUANTIZED WITH RESOLUTION $200\text{Nano-SEC} * C$ OR APPROXIMATELY $\sim 200 \text{ FEET}$. ($1\text{Nano-SEC} \sim 1\text{FOOT} @ C$)

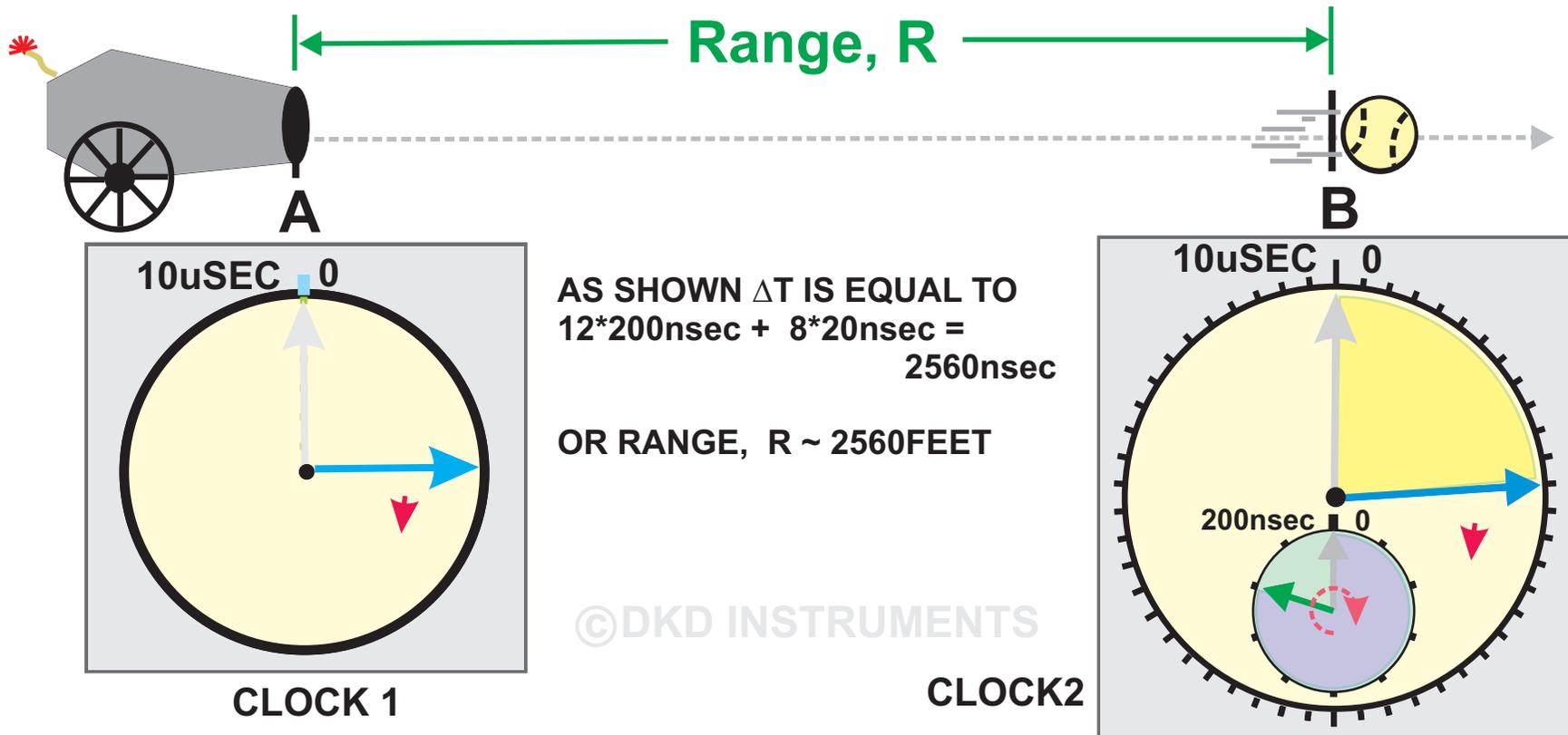


AN INCREASE IN RANGE RESOLUTION WITH SPEED OF LIGHT BASEBALL

WE CAN INCREASE OUR RANGE RESOLUTION BY INCREASING THE RESOLUTION OF CLOCK2.

WE ADD A SECOND DIGITAL DIAL TO CLOCK2 THAT HAS 10 TICKS IN 200Nano-SEC. FOR EVERY CYCLE OF THIS NEW DIAL THE 10uSEC DIAL ADVANCES BY ONE TICK. BOTH THE DIALS OF CLOCK2 READ ZERO WHEN CLOCK1 READS ZERO.

OUR TIME AND RANGE RESOLUTION IS NOW INCREASED BY A FACTOR OF TEN TO 20Nano-SEC AND 20 FEET RESPECTIVELY.



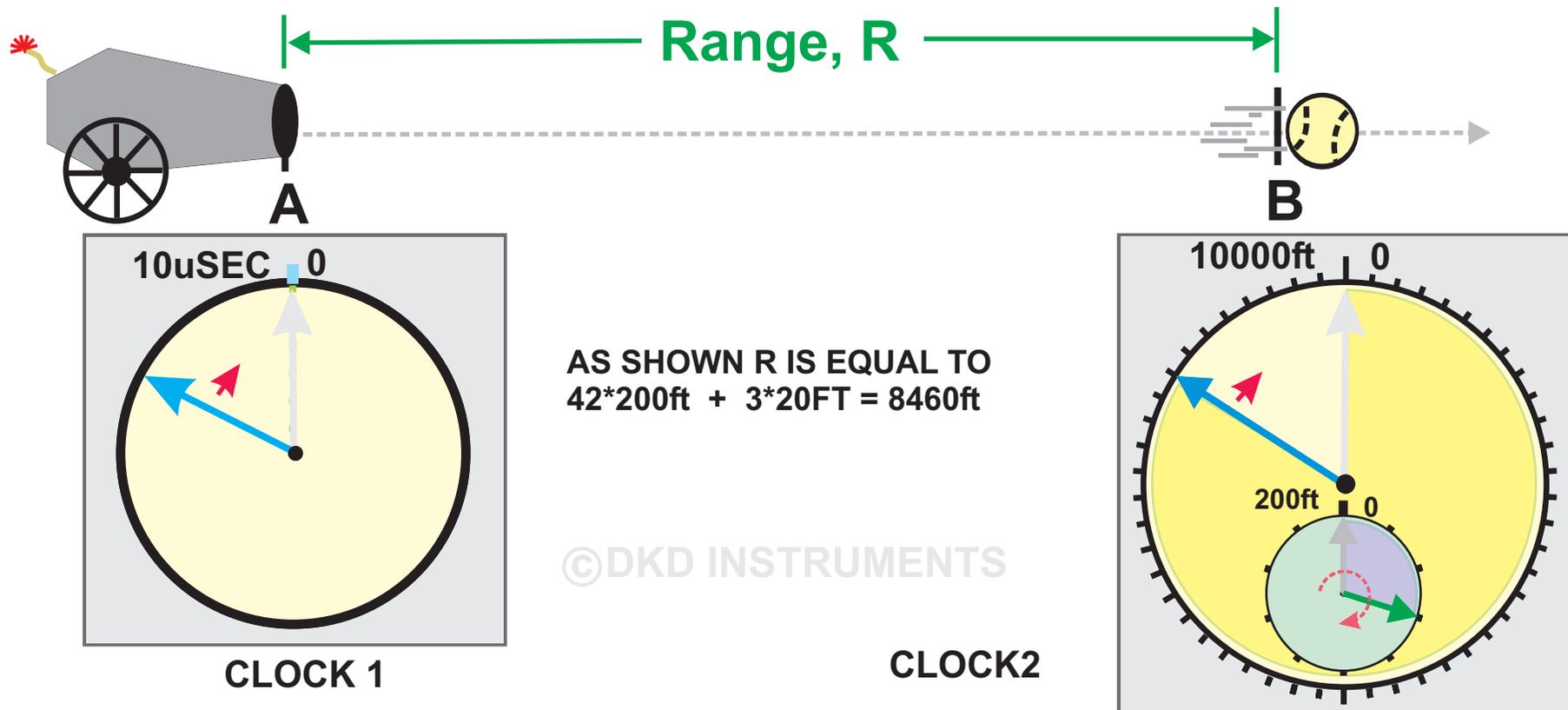
RE-SCALE CLOCK2 TO READ DIRECTLY IN FEET

WE CAN RESCALE CLOCK2 TO READ DIRECTLY IN FEET BY MULTIPLYING BY C, OR JUST USE THE APPROXIMATION THAT OUR BASEBALL AT SPEED C TRAVELS 1FOOT IN ABOUT 1nano-sec

WE CAN SEE NOW THE MAXIMUM RANGE WE CAN RECORD IS 10000feet. THE SMALLEST RANGE WE CAN RESOLVE IS 20feet.

IF WE WISH TO MEASURE LARGER DISTANCES WE NEED TO ADD DIALS ABOVE THE 10000ft DIAL. FOR SMALLER RANGES ADD DIALS BELOW THE 200ft DIAL.

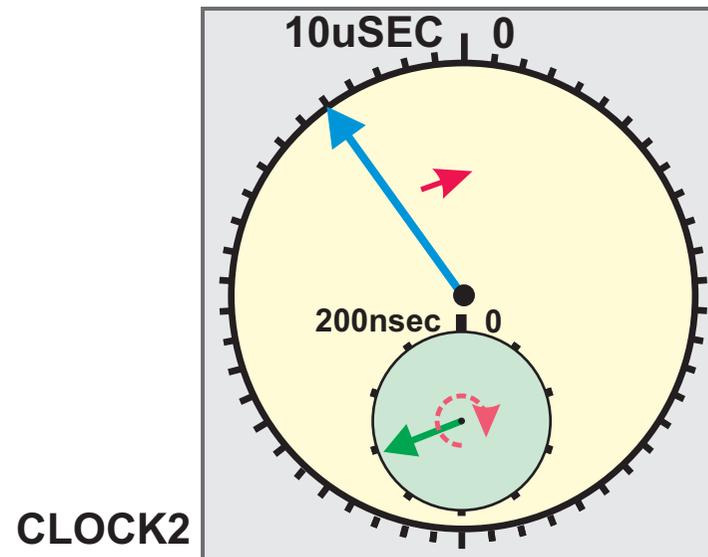
WE CAN SEE NOW THAT CLOCK2 HAS TWO SCALES OF RANGE OR TIME



WE HAVE SHOWN HOW RANGE CAN BE MEASURED USING TWO SYNC'ED CLOCKS AND A CONSTANT SPEED BASEBALL.

WE ADDED A 10u_{sec} CLOCK DIAL WHEN WE NEEDED TO MEASURE A SPEED OF LIGHT BASEBALL. WE ALSO ADDED 200n_{sec} DIAL TO CLOCK2 TO INCREASE OUR RANGE RESOLUTION FOR THE HYPER-SPEED BASEBALL.

THE GPS SYSTEM USES VERY HIGH PRECISION CLOCKS THAT CAN MEASURE ON TIMES SCALES FROM SUB-NANO SECONDS TO SECONDS. ITS TIME TO INTRODUCE OUR MULTI DIAL, ROTARY CLOCK MODEL OF THE GPS L1 SIGNAL .



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A FIVE SCALE ROTARY CLOCK MODEL FOR GPS RECEIVERS, L1 SIGNAL

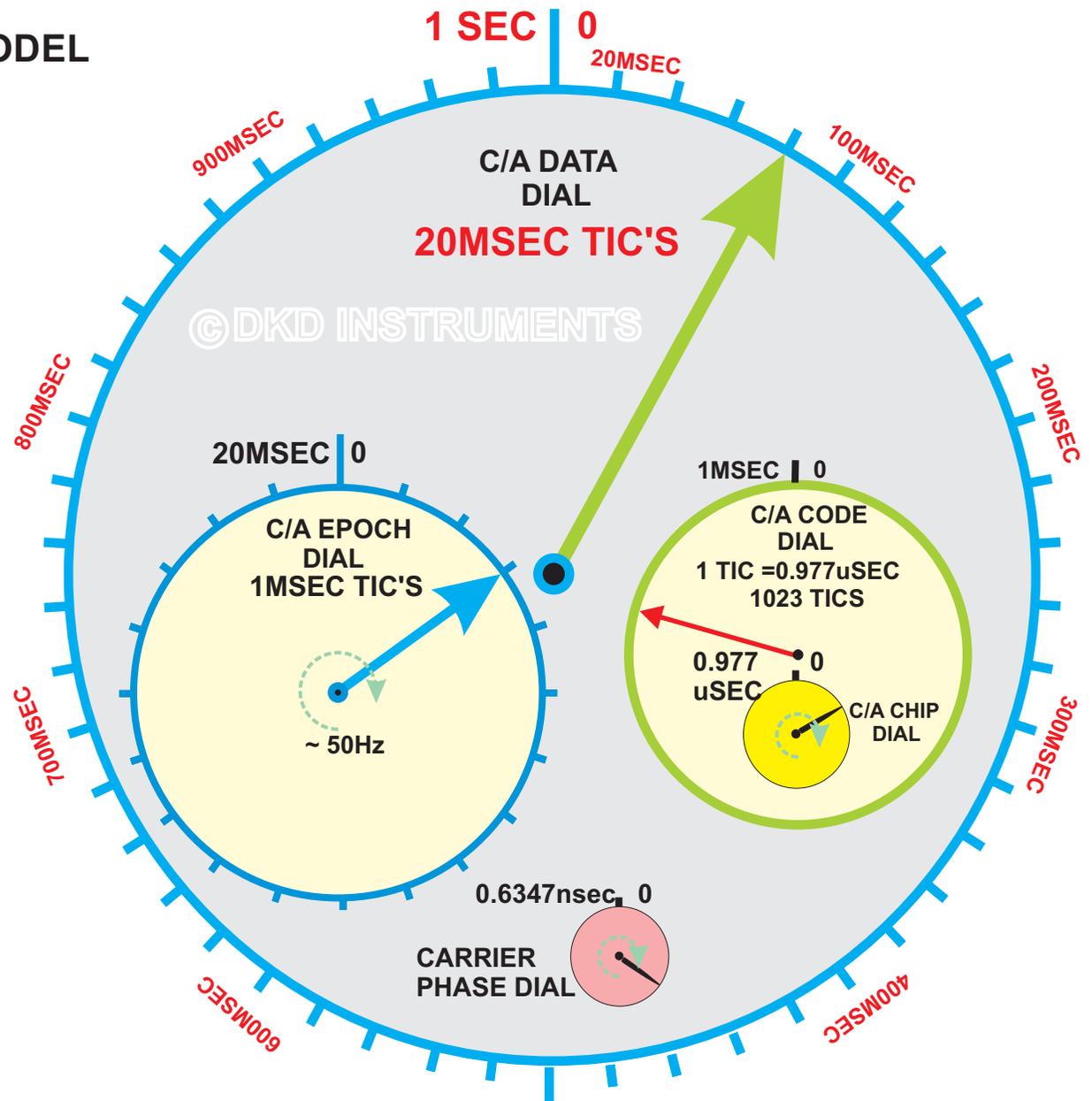
TO THE RIGHT IS OUR MODEL OF THE GPS CLOCK WE WILL USE IN THIS TEXT. THIS MODEL IS FOR THE L1 SIGNAL.

ITS A FIVE DIAL ROTARY TYPE CLOCK.

THE C/A CHIP DIAL AND THE CARRIER PHASE DIALS ARE ANALOG DIALS. THE OTHER DIALS ARE DIGITAL. ALL DIALS ROTATE CLOCKWISE.

THE MAX TIME THAT CAN BE MEASURE IS 1SEC.

THE MINIMUM TIME THAT IS RESOLVABLE IS APPROXIMATELY 1/100 OF THE CARRIER PHASE DIAL OR 6.347pico-sec



THE RATIOS OF THE DIALS ARE;

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- 50 CYCLES OF THE C/A EPOCH DIAL FOR ONE CYCLE OF THE 1 SECOND DIAL
- 20 CYCLES OF THE C/A CODE DIAL FOR ONE CYCLE OF THE C/A EPOCH DIAL
- 1023 CYCLES OF THE C/A CHIP DIAL FOR ONE CYCLE OF THE C/A CODE DIAL
- 1540 CYCLES OF THE CARRIER PHASE DIAL FOR ONE CYCLE OF THE C/A CHIP DIAL (IF LOCKED)

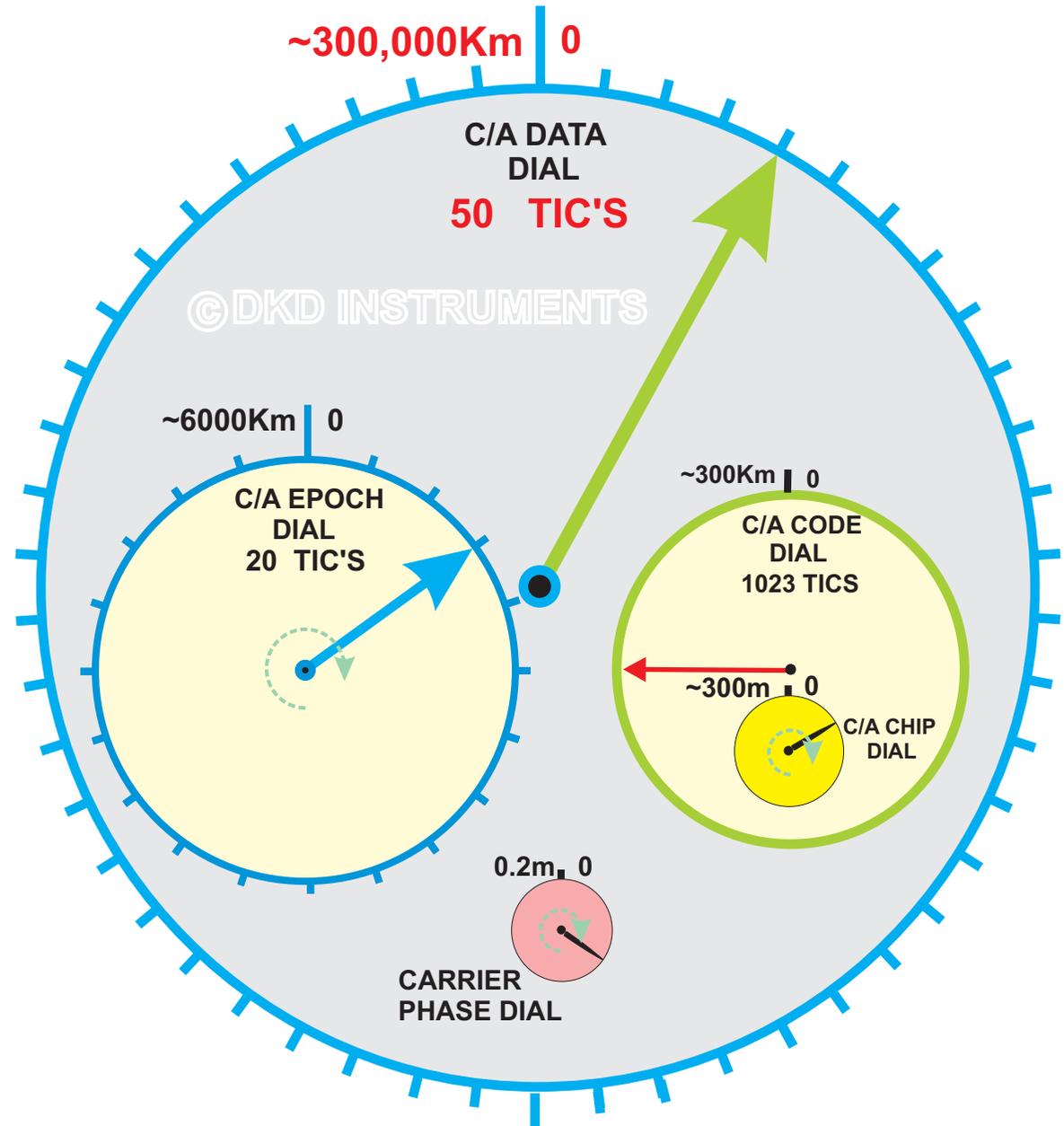
**GPS L1 ROTARY CLOCK RESCALED TO READ RANGE IN METERS (Approx Values Used)
OUR GPS CLOCK HAS MANY SCALES TO MEASURE RANGE WITH**

**BY SCALING OUR L1 CLOCK BY C
WE CAN EASILY SEE THAT THE
MAXIMUM RANGE IT CAN
MEASURE IS ~300,000Km**

**IF WE ASSUME THAT A
RESOLUTION 1/100 OF THE
CARRIER PHASE DIAL IS POSSIBLE
OUR FINEST RANGE RESOLUTION
IS ~ 0.002meters!**

**TYPICAL RANGES TO GPS
SATELLITES VARY BETWEEN
20,000Km TO 26,000Km. THIS IS
LESS THAN OUR MAXIMUM RANGE
AS NOTED ABOVE.**

**THE TIC SIZES ARE;
C/A DATA DIAL TIC: ~ 6000Km
C/A EPOCH DIAL TIC: ~ 300Km
C/A CODE DIAL TIC: ~300m
C/A CHIP DIAL RESOLUTION: 3m*
CARRIER DIAL RESOLUTION: 2mm*
* assumes 1/100 cycle resolution**



**AS CLOCK2 IN OUR HYPER SPEED BASEBALL THE RANGE AS SHOWN IS;
 $4*6000\text{Km} + 3*300\text{Km} + 750*300\text{m} + 0.3*300\text{m} + 0.7*0.2\text{m} = 25125100.14 \text{ METERS}$**

WHAT IS A "CODE PHASE MEASUREMENT" ?

A VERY COMMON TERM IS GPS TEXTS AND CONVERSATION IS THE TERM "CODE PHASE MEASUREMENT" UNFORTUNATELY THIS TERM IS RARELY DEFINED.

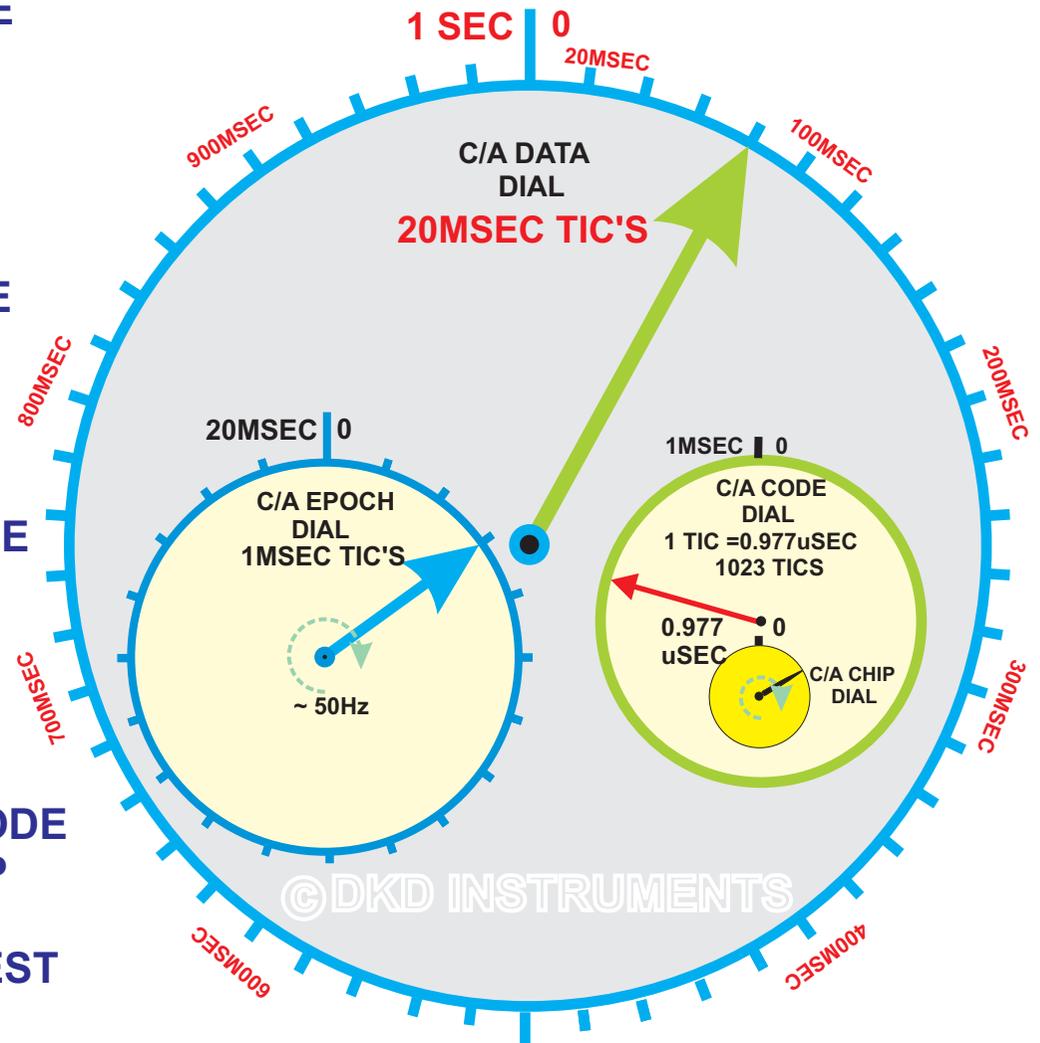
A COMMON MEANING IS MAKING A RANGE MEASUREMENTS USING JUST THE C/A CODE DIAL AND THE C/A CHIP DIAL, NO OTHER DIALS.

SOMETIMES IT MEANS USING ALL DIALS EXCEPT THE CARRIER PHASE DIAL IN RANGE MEASUREMENTS.

IN THIS TEXT WE SHALL USE THE LATTER MEANING, UP TO THE 1 SECOND DIAL.

USING THAT DEFINITION WE SEE THAT "CODE PHASE MEASUREMENTS" CAN MEASURE UP TO 300,000Km WITH A RESOLUTION OF APPROXIMATELY 3 METERS AS THE SMALLEST DIAL IS NOW THE C/A CHIP DIAL.

FOR CIVILIAN GRADE RECEIVERS THAT RESOLUTION IN RANGE TO THE SATELLITE IS ENTIRELY ADEQUATE, HENCE CIVILIAN POSITION ACCURACY IS AT BEST 3 TO 10 METERS.



WHAT IS A "CARRIER PHASE MEASUREMENT" ?

ANOTHER VERY COMMON TERM IN GPS TEXTS AND CONVERSATION IS THE TERM "CARRIER PHASE MEASUREMENT"

CARRIER PHASE MEASUREMENTS ARE NEARLY ALWAYS RANGE (OR RANGE RATE) MEASUREMENTS TAKEN JUST FROM THE CARRIER DIAL ONLY.

RANGE MEASUREMENTS FROM THE CARRIER DIAL HAVE A MAXIMUM DISTANCE OF ~ 0.2 METERS AND A TYPICAL RESOLUTION OF A FEW MILLIMETERS.

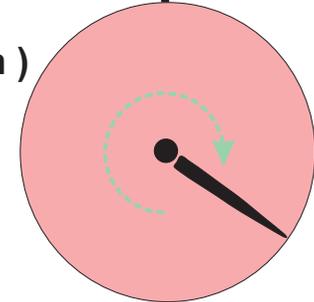
MOST HANDHELD GPS RECEIVERS DO NOT MAKE CARRIER PHASE RANGE MEASUREMENTS DUE TO THE COMPLEXITIES OF THESE TYPE OF MEASUREMENTS. IN PARTICULAR THE RANGE TO THE SATELLITE CAN ONLY BE EXPRESSED AS;

$RANGE\ TO\ SV = CYCLE\ FRACTION(0.2m) + N * 0.2m$

WHERE FINDING " N " IS A NON TRIVIAL TASK !

CARRIER PHASE RANGE MEASUREMENTS ARE TYPICALLY FOUND ONLY IN PRECISION , MULTI- BAND RECEIVERS.

0.6347nsec, 0
(OR scaled to
RANGE ~ 0.2m)



CARRIER PHASE DIAL

A MULTISCALE TAPE MEASURE AND AMBIGUOUS DISTANCE

BELOW IS A SMALL SECTION OF A VERY LONG TAPE MEASURE. IS CONSTRUCTED OF SUCCESSIVELY SMALLER SCALES. THE LARGEST SCALE IS FROM 0 TO 100KM, THEN 0 TO 1000METERS, 0 TO 10 DECIMETERS AND FINALLY 0 TO 10 CENTIMETERS.

TO READ THE DISTANCE INDICATED WE TAKE THE SMALLER READING OFF EACH SCALE AND ADD THEM UP;

$$\text{DISTANCE} \sim 38\text{Km} + 1\text{m} + 1\text{decimeter} + 3\text{centimeter} \Rightarrow 38,001.13 \text{ METERS}$$

WE CAN ALSO READ AN APPROXIMATE AND **AMBIGUOUS** DISTANCE OFF EACH SCALE, EACH WITH THE RESOLUTION OF THAT SCALE;

KILOMETER SCALE, 1Km RESOLUTION: $\text{DISTANCE} \sim 38\text{Km} + N \cdot 100\text{Km}$, N INTEGER

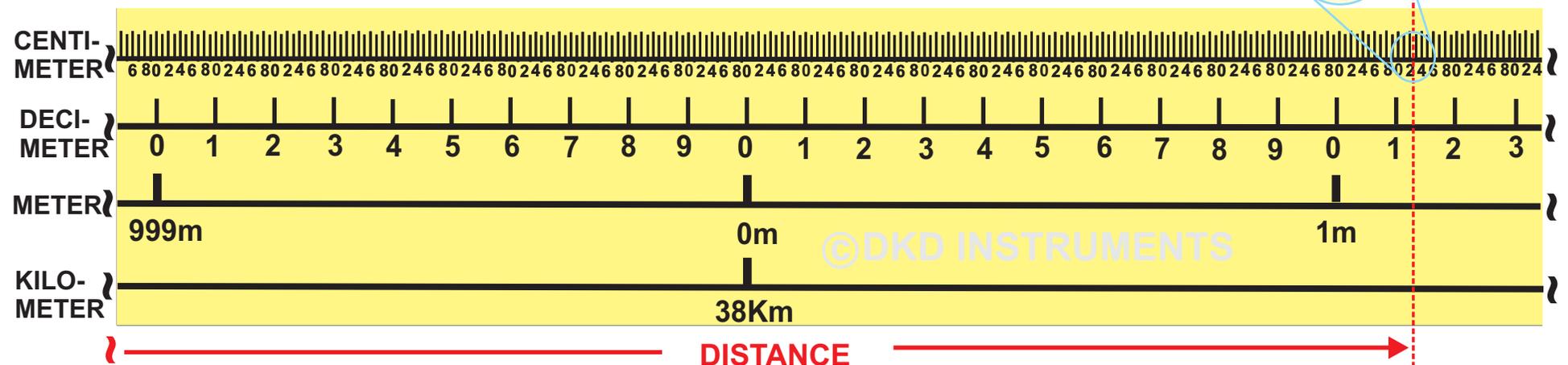
METER SCALE, 1m RESOLUTION: $\text{DISTANCE} \sim 1\text{M} + K \cdot 1000\text{m}$, K INTEGER

DECIMETER SCALE, 0.1m RESOLUTION: $\text{DISTANCE} \sim 1\text{decimeter} + L \cdot 10\text{decimeters}$, L INTEGER

CENTIMETER SCALE, 0.01m RESOLUTION: $\text{DISTANCE} \sim 3\text{centimeter} + J \cdot 10\text{centimeters}$, J INTEGER

IF THE DISTANCE IS KNOWN WE CAN FIND THE INTEGERS N,K,L & J ABOVE. TO FIND J FOR EXAMPLE, FIRST SUBTRACT OFF THE 3 CENTIMETER READING FROM 38,001.13 AND THEN DIVIDE BY THE RESOLUTION, $J = 3800100$.

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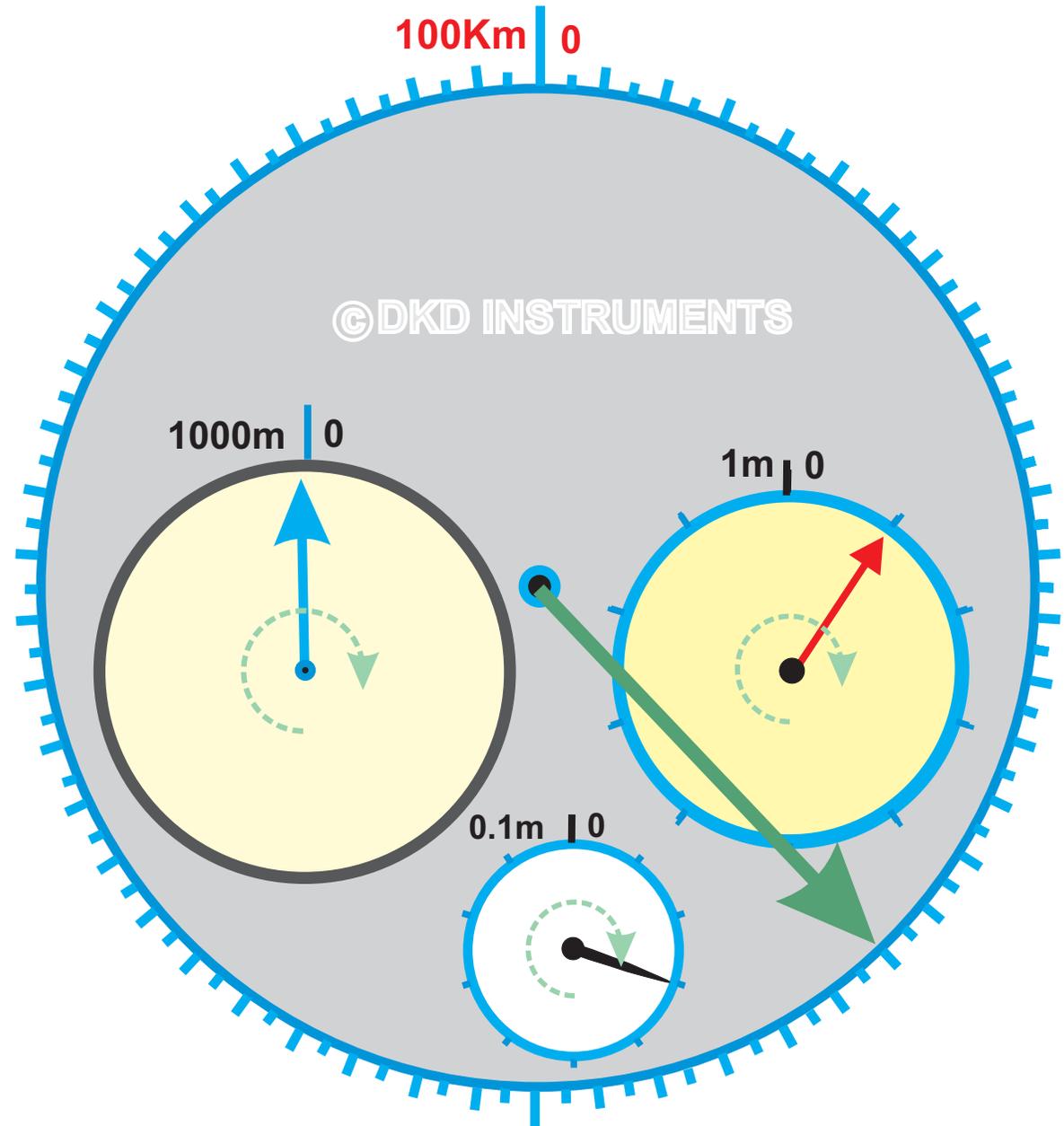
A ROTARY CLOCK EQUIVALENT OF THE MULTI-SCALE TAPE MEASURE

TO THE LEFT WE SEE A ROTARY CLOCK SCALED IN METERS. THE DIALS ARE SETUP WITH SAME RATIOS AS OUR MULTI-SCALE TAPE MEASURE.

THE *PHASE STATE* INDICATED IS THE SAME DISTANCE AS WE READ OFF THE TAPE MEASURE.

ALTHOUGH OUR GPS L1 CLOCK HAS DIFFERENT DIAL SCALES WE CAN SEE THAT IT IS OF THE SAME TYPE AS OUR CLOCK SHOWN HERE. THIS OF COURSE MEANS THE L1 GPS CLOCK IS ALSO A MULTI-SCALE TAPE MEASURE WE CAN USE TO MEASURE THE DISTANCE FROM THE RECEIVER TO THE SATELLITE. LASTLY WE SEE NOW THAT EACH SCALE OR DIAL OF THE GPS L1 CLOCK MUST HAVE AN AMBIGUOUS RANGE.

FOR SPEED OF LIGHT TRANSMISSION WE CAN RE-SCALE THE CLOCK TO READ TIME BY DIVIDING EACH DIAL SCALE BY SPEED OF LIGHT, c . THE 0.1METER DIAL WOULD THEN HAVE A PERIOD OF 0.3356409nsec OR A RATE OF 2,997,924,580Hz



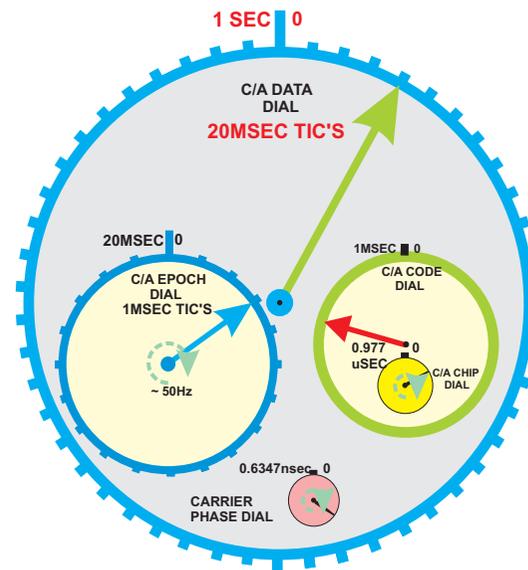
GPS L1 CLOCK SCALES: TIME , RANGE RESOLUTION AND RANGE AMBIGUITY

THE TABLE BELOW DETAILS THE TIME AND RANGE RESOLUTIONS FOR THE GPS L1 CLOCK. THE INTEGERS REPRESENT THE NUMBER OF TIMES THE UNAMBIGUOUS RANGE IS PRESENT IN TWO RANGES: THE MINIMUM SATELLITE RANGE AND THE MAXIMUM SATELLITE RANGE FOR A USER AT OR NEAR THE EARTH'S SURFACE.

DIAL	RESOL. TIME	RESOL. METERS	UNAMBIGUOUS RANGE, METERS	INTEGER @ MIN. SATELLITE RANGE.	INTEGER @ MAX. SATELLITE RANGE.
CARRIER PHASE	~ 6.347pico-sec	~ 0.002	0.1902782	105109256	134503059
C/A CHIP	~ 10nsec	~ 3	292.897231	68283	87379
C/A CODE	0.977usec	292.897231	299,792.458	67	85
C/A EPOCH	1msec	299,792.458	5,995,849.16	3	4
C/A DATA (1SEC)	20msec	5,995,849.16	299,792,458	N/A	N/A

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MIN SAT RANGE 20,000Km
MAX SAT RANGE 25,593Km

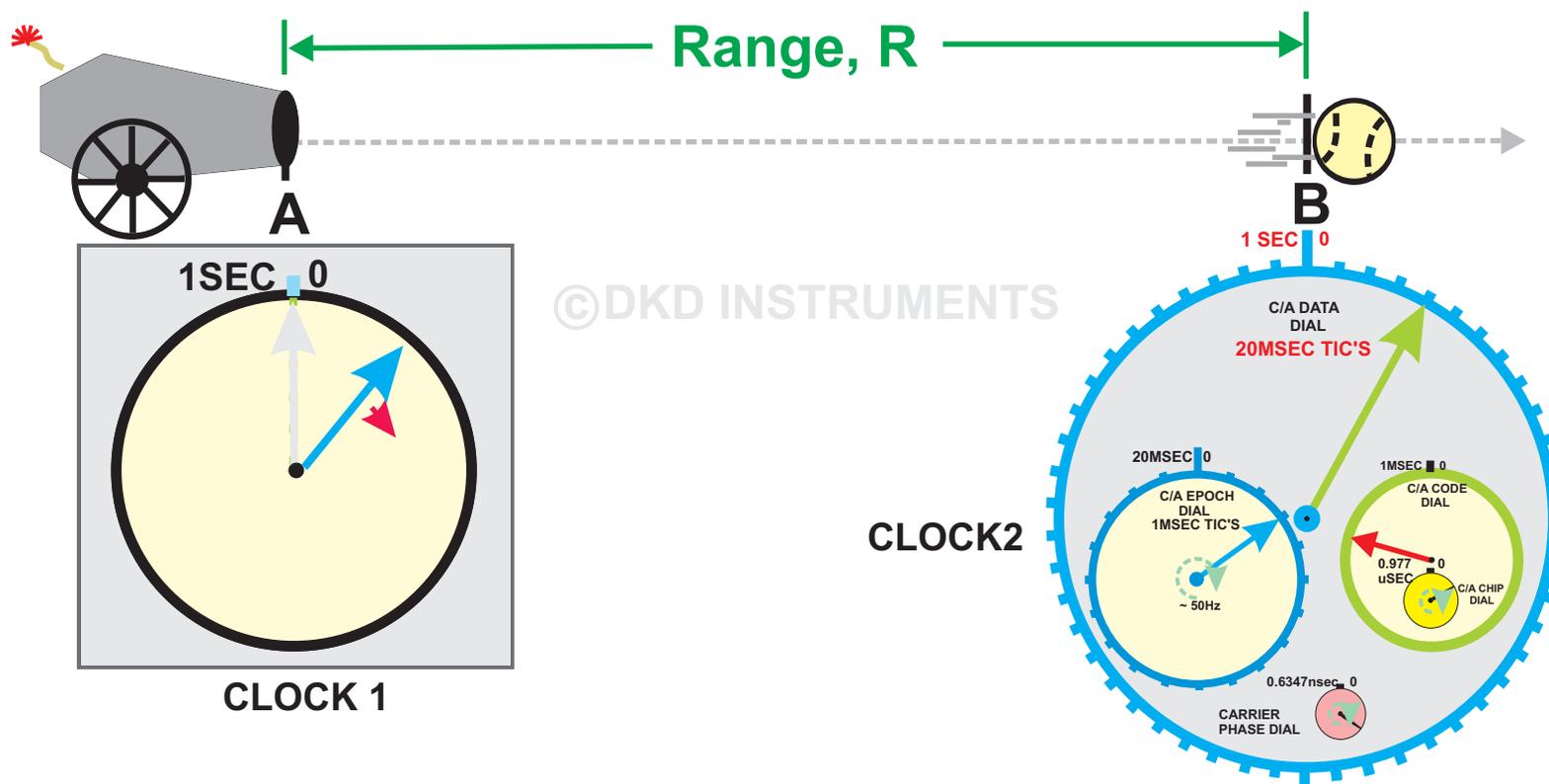


THE NEED FOR A BETTER RANGE MEASUREMENT MODEL

WE COULD NOW INSERT OUR NEW GPS CLOCK INTO OUR HYPER SPEED BASEBALL MODEL AS SHOWN BELOW. BUT AT THIS POINT WE NEED A BETTER MODEL TO INVESTIGATE MEASURING DISTANCE USING TWO CLOCKS.

IN PARTICULAR THE CANNON IS ONLY SENDING A SHOT ONCE PER SECOND. WE COULD MAKE CLOCK1 THE SAME AS CLOCK2 AND SHOOT A BASEBALL OFF WHENEVER ANY DIAL HITS ZERO. IN PRINCIPLE THAT WOULD WORK BUT ITS A BIT MESSY.

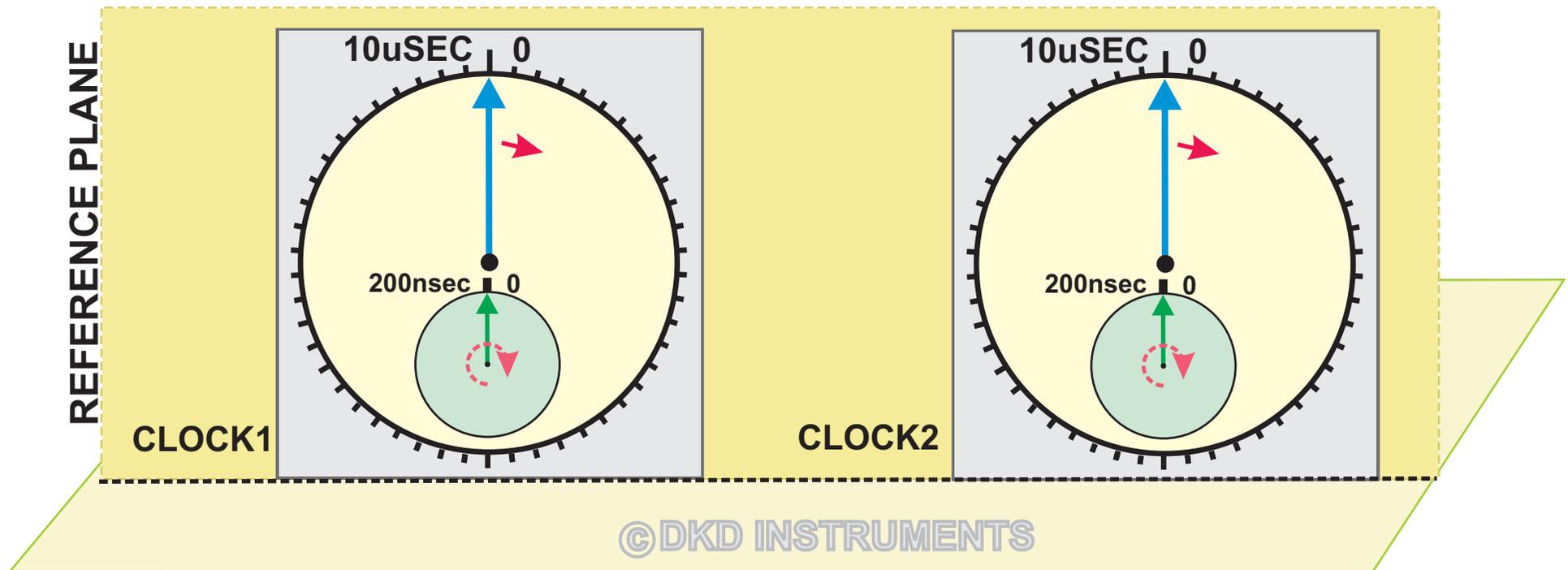
WE NEED A MORE ELEGANT MODEL , ONE THAT REFLECTS A CLOSER MATCH TO THE PHYSICS THAT IS OCCURRING.



A NEW MODEL FOR MEASURING RANGE WITH TWO CLOCKS

BELOW WE HAVE TWO CLOCKS RUNNING AT THE SAME RATE AND IN PHASE(ALWAYS). THE ANALOG DIAL CYCLES IN 200nanoSEC , THE DIGITAL DIAL CYCLES IN 10uSEC. THE TIC SIZE OF THE 10uSEC DIAL IS 200nanoSEC OR FOR EVERY 50 CYCLES OF THE 200nanoSEC DIAL ONE CYCLE OF THE 10uSEC DIAL. WE WILL ASSUME THE RESOLUTION OF THE 200NSEC DIAL IS APPROXIMATELY 1/100 CYCLE.

AS SHOWN THE CLOCKS ARE ON THE SAME REFERENCE PLANE, THAT IS THERE IS NO DISTANCE DIFFERENCE WITH RESPECT TO THIS PLANE FOR CLOCK1 OR CLOCK2. AN OBSERVER AT THE REFERENCE PLANE WOULD SEE THE IMAGE BELOW WHEN DIAL IS AT ZERO.

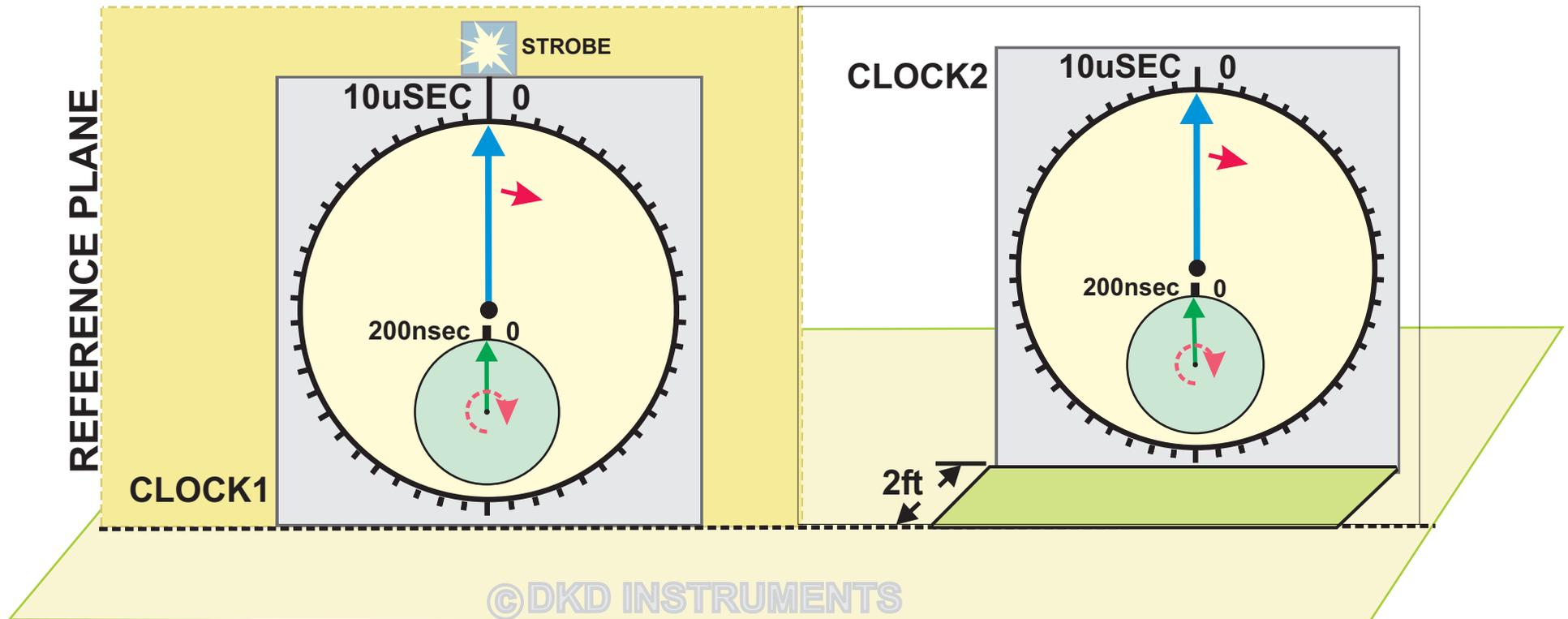


CLOCKS ARE NOT ON THE SAME REFERENCE PLANE

NOW WE MOVE CLOCK2 BACK AWAY FROM THE REFERENCE PLANE BY 2ft.

THE IMAGE SHOWN IS A SNAP SHOT TAKEN (at strobe instant) WHEN CLOCK1 DIALS ARE AT ZERO. IN ADDITION THE IMAGE IS THAT SEEN BY AN OBSERVER AT THE REFERENCE PLANE. ASSUME STROBE USES A LIGHT WITH INFINITE SPEED.

THE IMAGE OF OUR TWO CLOCKS @ THE REFERENCE PLANE IS THE SAME AS BEFORE , RIGHT?



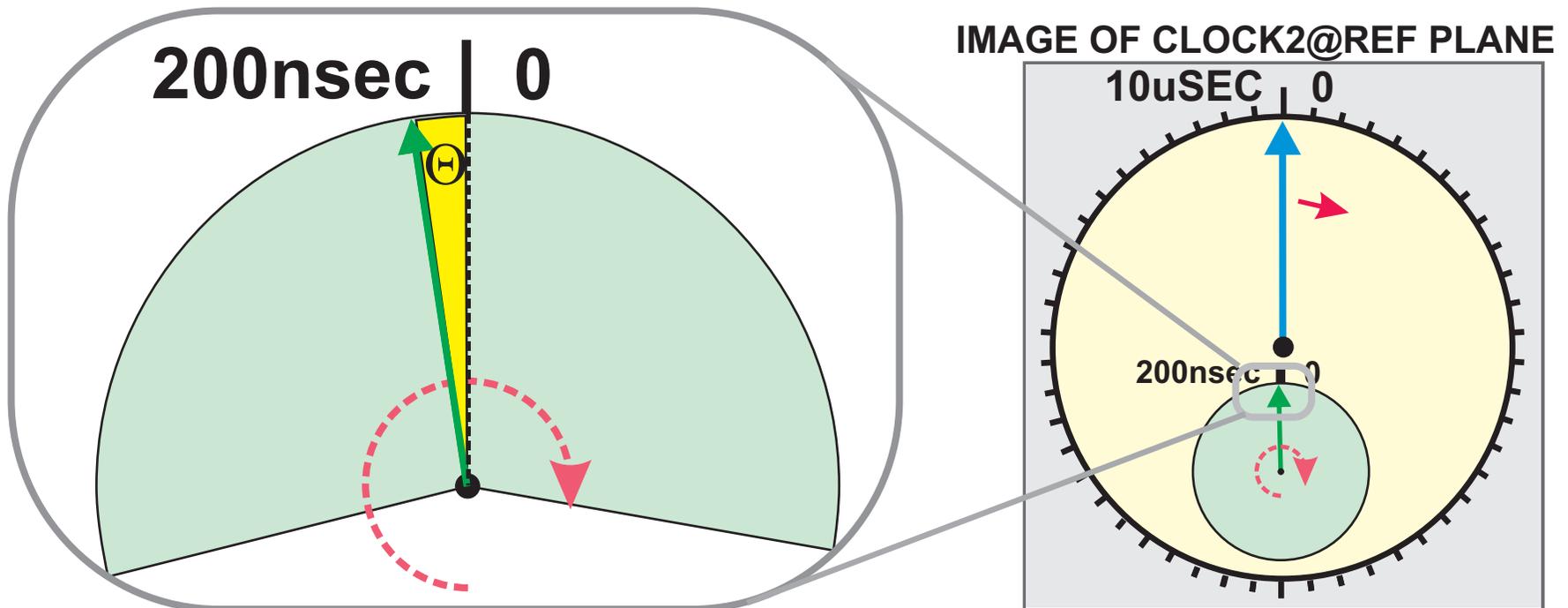
CLOSE UP OF 200nanoSEC DIAL FOR 2 FEET OF DELAY

NOT QUITE. ON CLOSE EXAMINATION OF CLOCK2 WE SEE THAT THE 200nanoSEC DIAL HAS A PHASE OFFSET OF ~ 3.6 DEGREES FROM ZERO POINT..

THIS OCCURS BECAUSE AT THE REFERENCE PLANE THE IMAGE OF CLOCK2 ARRIVED AFTER A DELAY OF ~ 2 NSEC. (1nano-SEC per foot @ C)

IN OTHER WORDS AT THE REFERENCE PLANE WE ARE SEEING CLOCK2 AS IT WAS IN THE PAST. WITH OUR 200nanoSEC DIAL WE CAN JUST BARELY MEASURE IT AS OUR RESOLUTION FOR THE 200nanoSEC DIAL IS ~ 3.6 DEGREES.

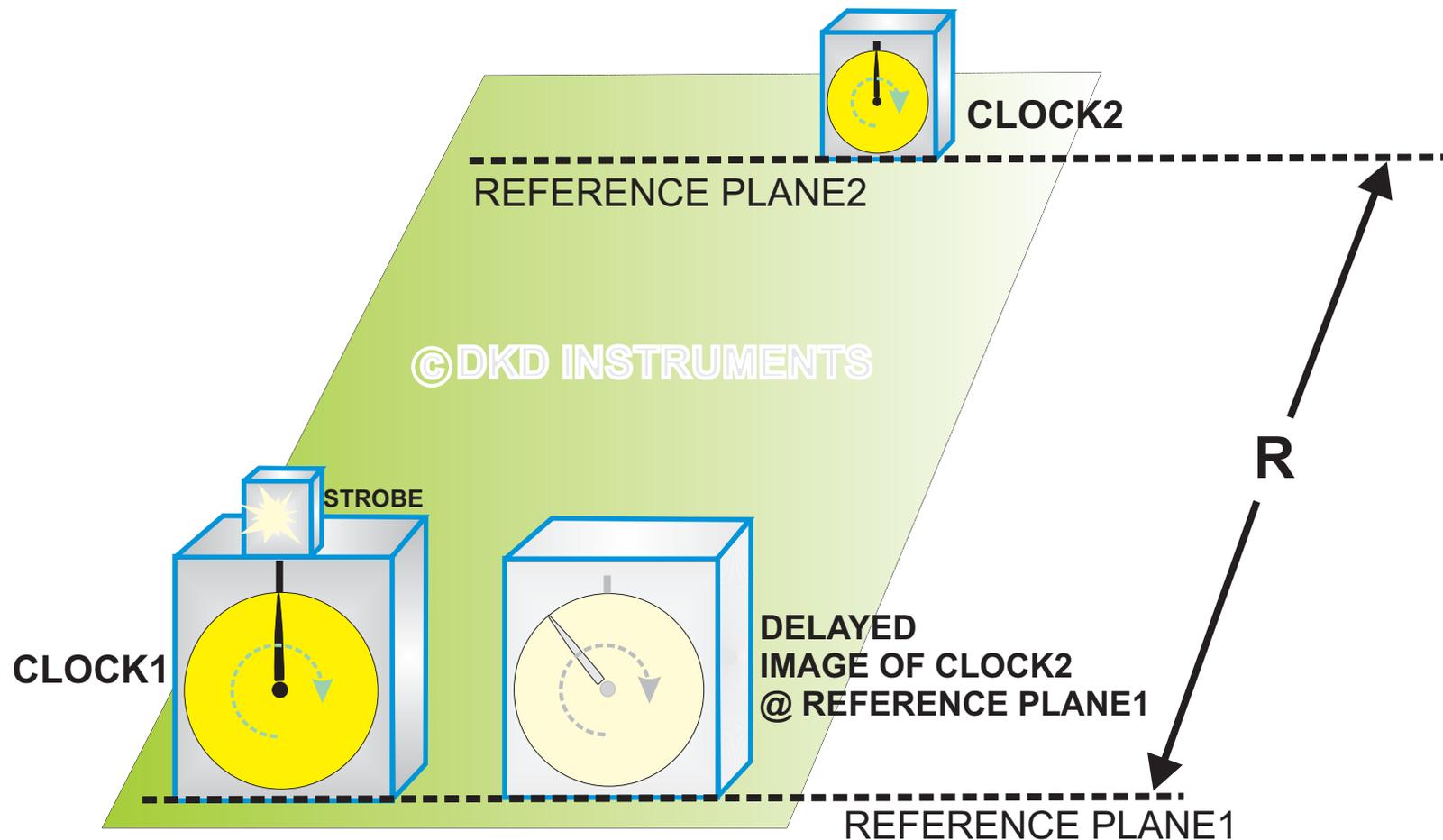
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TO HELP UNDERSTAND WHAT IS HAPPENING WE NEED A BETTER PICTURE THAN OUR "SCHEMATIC TYPE" AS JUST PRESENTED.

BELOW WE SEE THAT AN OBSERVER AT REFERENCE PLANE1 SEES THE DELAYED IMAGE OF CLOCK2. DELAY = R / C

AN OBSERVER AT REFERENCE PLANE2 SEES CLOCK2 WITH NO PHASE OFFSET wrt CLOCK1. (all phases recorded at Clock1 zero mark w/strobe)



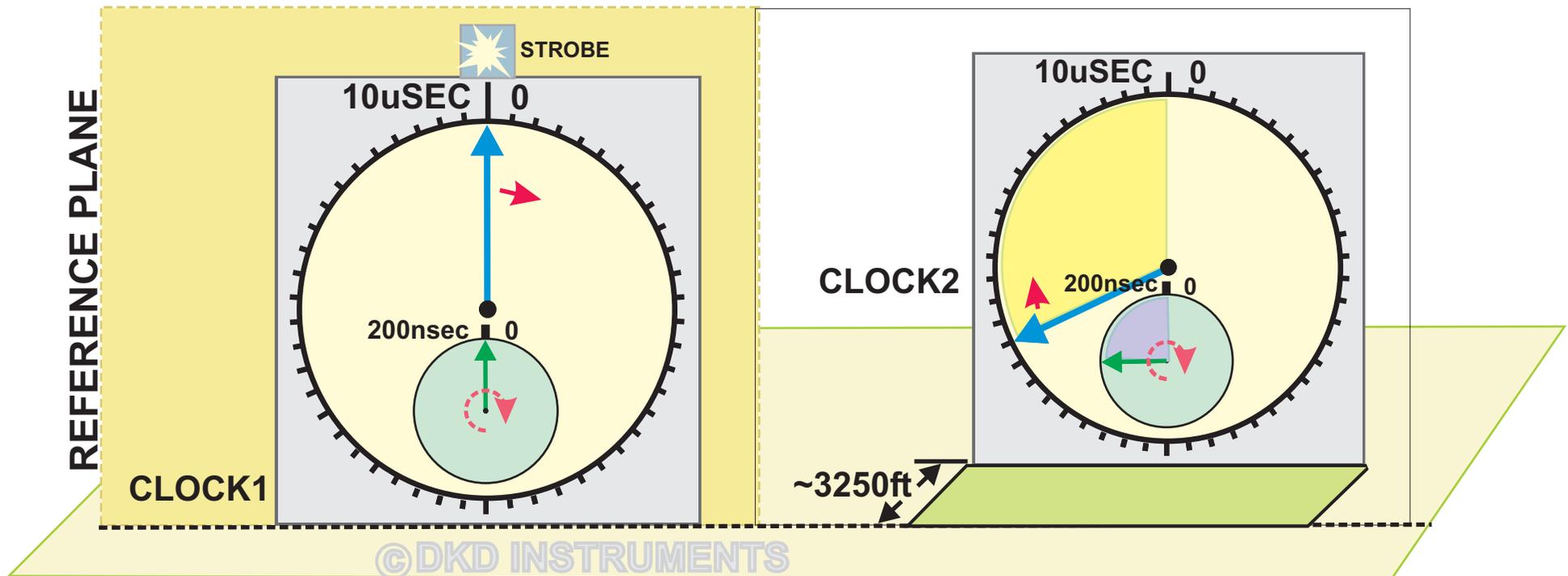
CLOCK2 OFFSET BY ~3250ft FROM REFERENCE PLANE

NOW WE MOVE CLOCK2 BACK AWAY FROM THE REFERENCE PLANE BY 3250ft.

AS BEFORE THE IMAGE SHOWN IS A SNAP SHOT TAKEN AT THE REFERENCE PLANE WHEN CLOCK1 DIALS ARE AT ZERO. AN OBSERVER AT THE REFERENCE PLANE WOULD SEE THE IMAGE BELOW AT THAT INSTANT .

WE CAN NOW CLEARLY SEE THAT CLOCK2 HAS A PHASE OFFSET WITH RESPECT TO CLOCK1.

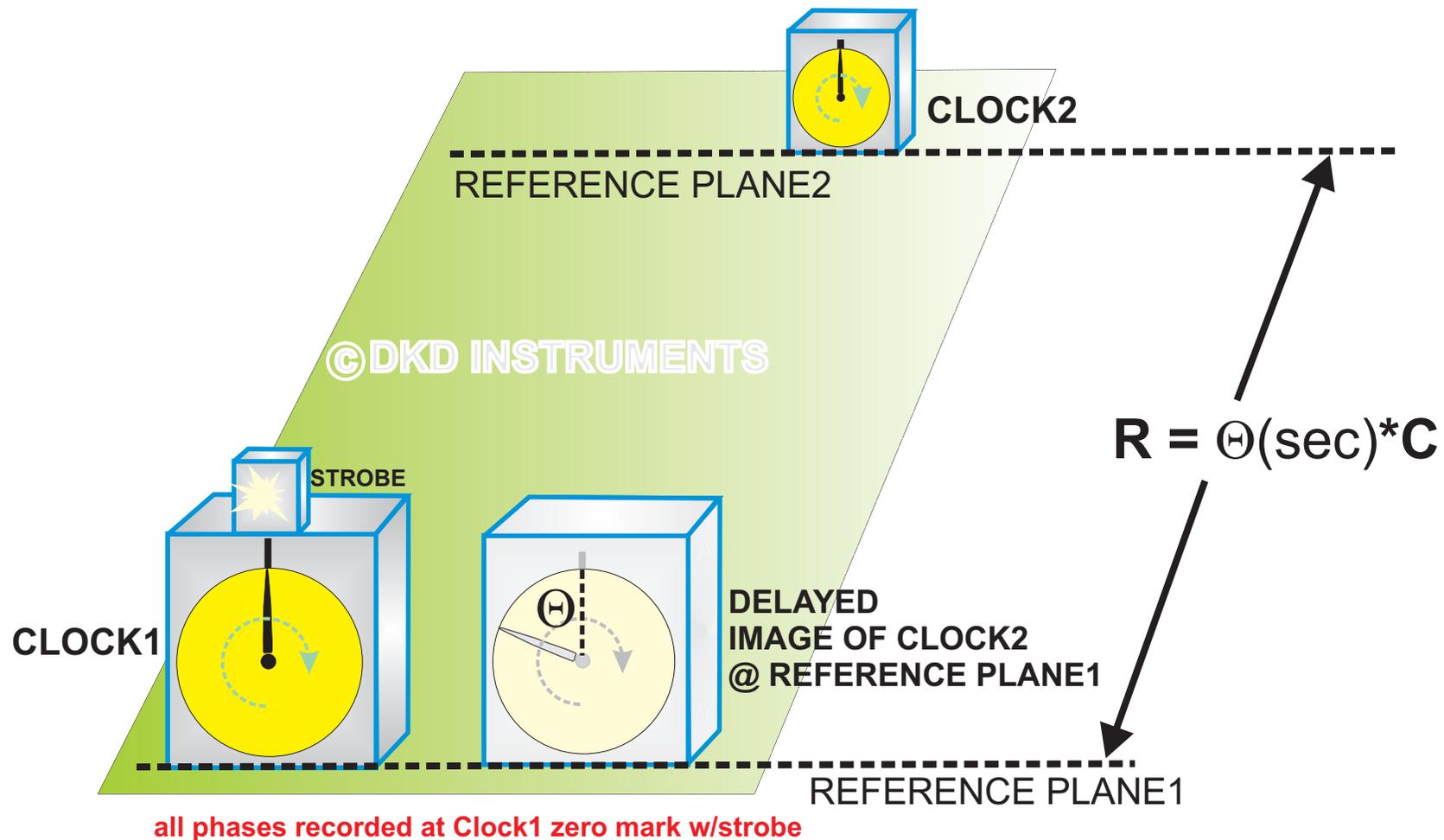
THE PHASE OFFSET IS MEASURED COUNTER-CLOCK WISE FROM ZERO POINT IS $16 \times 200\text{NSEC} + 0.25 \times 200\text{NSEC} \Rightarrow 3250\text{NSEC}$ OR AS A RANGE ~3250ft.



all phases recorded at Clock1 zero mark w/strobe

WE CAN NOW MAKE THE STATEMENT THAT AN OBSERVED PHASE DIFFERENCE AT REFERENCE PLANE 1 , Θ , BETWEEN CLOCK1 AND CLOCK2, IS A MEASURE OF THE RANGE BETWEEN CLOCK1 AND CLOCK2.

WE CAN CONVERT THIS PHASE TO UNITS OF DISTANCE BY CONVERTING Θ TO UNITS OF TIME (IN UNITS OF SECONDS TYPICALLY) AND THEN USING C TO CONVERT TO DISTANCE.



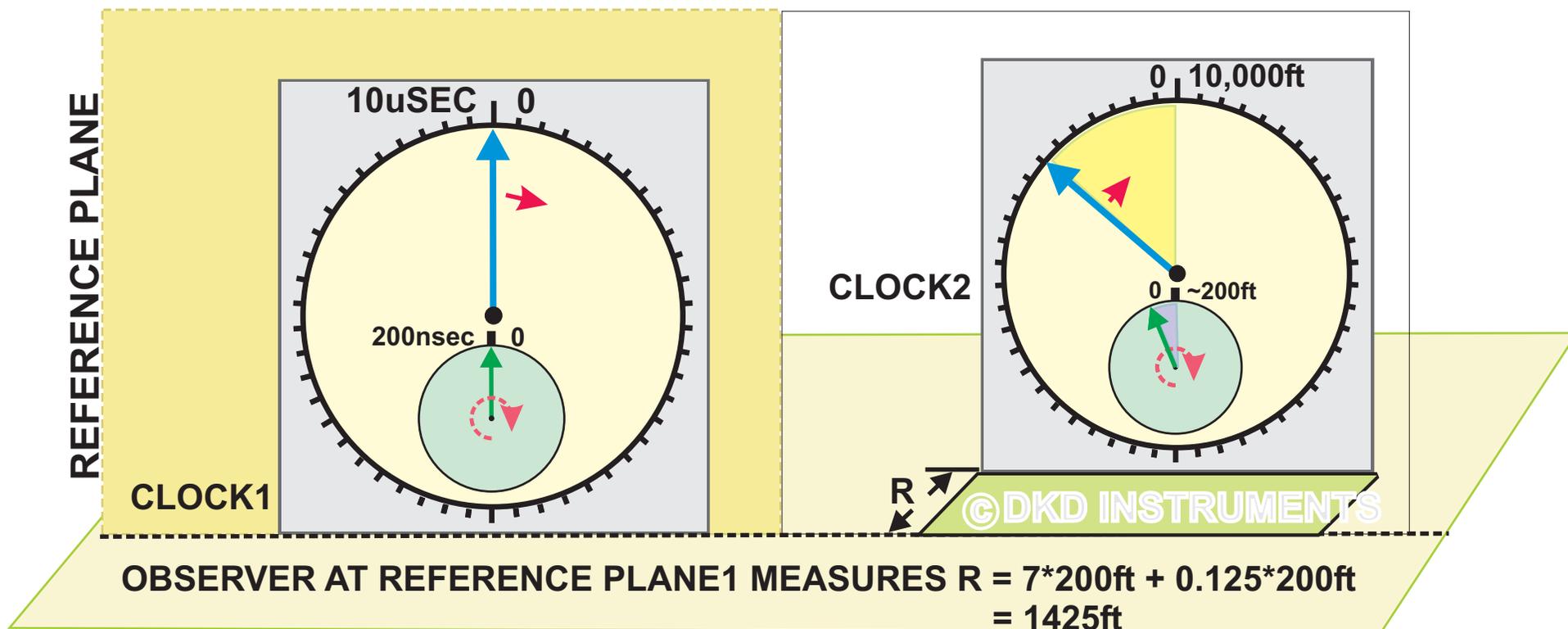
RESCALE CLOCK2 TO READ RANGE DIRECTLY

WE CAN RESCALE CLOCK2 AS BEFORE TO READ OUT RANGE DIRECTLY. AS WE HAVE SEEN THERE IS A REVERSAL IN THE WAY THE PHASE DIFFERENCE IS READ FROM THE DIAL COMPARED WITH OUR EARLIER HYPER SPEED BASEBALL MODEL.

THIS REVERSAL REFLECTS THE REVERSAL OF *TIME SENT AND TIME RECEIVED*. IN OUR PREVIOUS MODEL CLOCK1 WAS TIME SENT AND CLOCK2 WAS TIME RECEIVED. NOW ITS THE OPPOSITE. *THIS MEANS THAT THE TIME AN OBSERVER SEES AT REFERENCE PLANE1 ON CLOCK2 IS THE TIME SENT.*

TO ACCOMMODATE THIS REVERSAL WE NEED TO REVERSE THE MARKINGS ON CLOCK2 DIAL AND THEN SCALE BY C, SEE BELOW.

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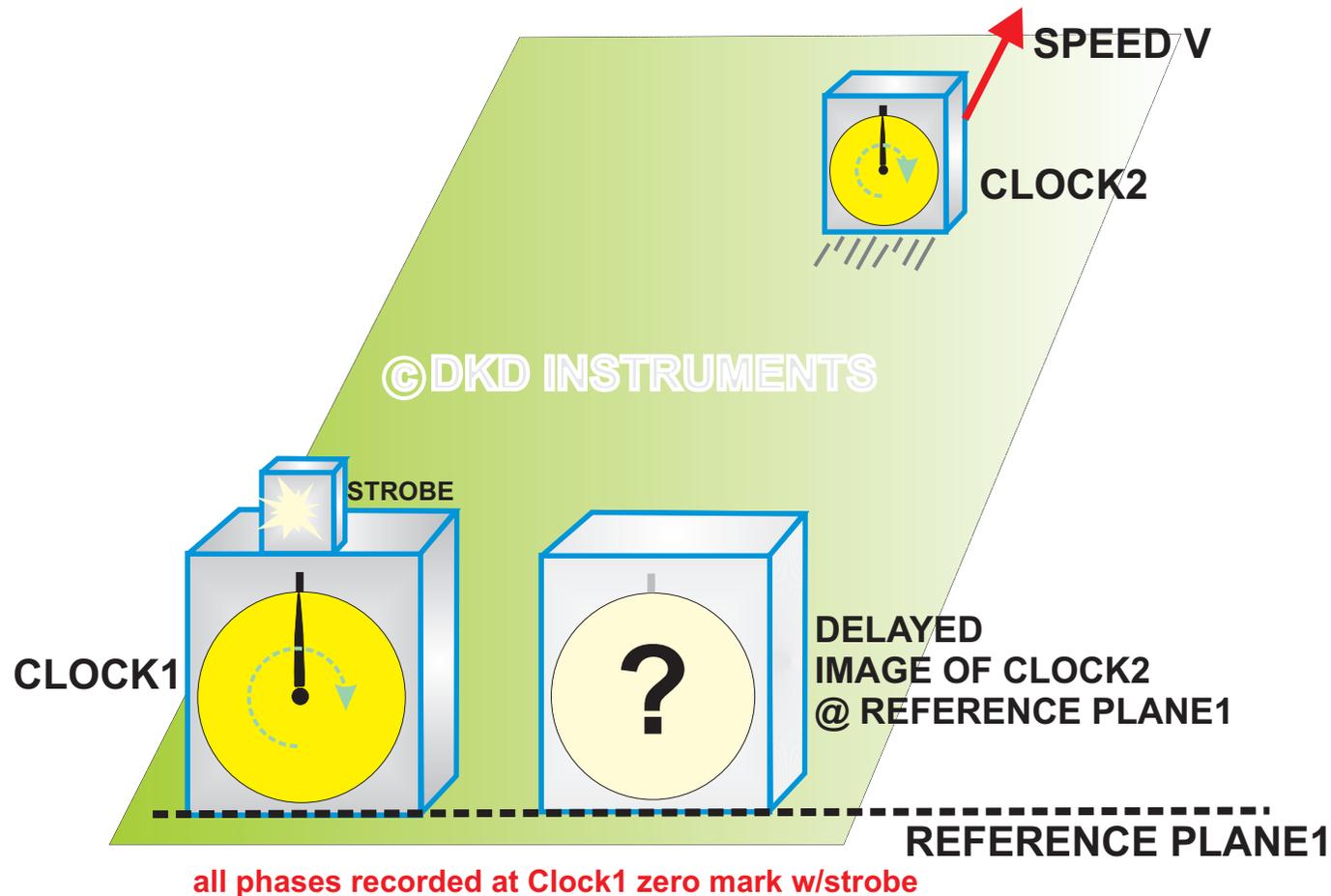


MEASURING THE SPEED OF A MOVING CLOCK

OUR NEW MODEL HAS GIVEN US A MORE ACCURATE REPRESENTATION OF HOW WE CAN USE TWO (SYNC'ED) CLOCKS TO MEASURE A FIXED OR STATIC RANGE.

BUT WHAT IF CLOCK2 IS MOVING AWAY FROM CLOCK1 AT A CONSTANT VELOCITY, V , AS SHOWN BELOW?

WHAT WOULD AN OBSERVER AT REFERENCE PLANE1 SEE ON CLOCK2?



TO ANSWER THIS QUESTION IMAGINE A SERIES OF SNAP-SHOTS OF THE IMAGE OF CLOCK2 AT REFERENCE PLANE1. THE SNAP-SHOTS ARE TAKEN EVERY TIME CLOCK1 HITS ZERO. RATHER THAN SHOW EACH SNAP SHOT WE OVERLAY THEM ON EACH OTHER AS SHOWN BELOW.

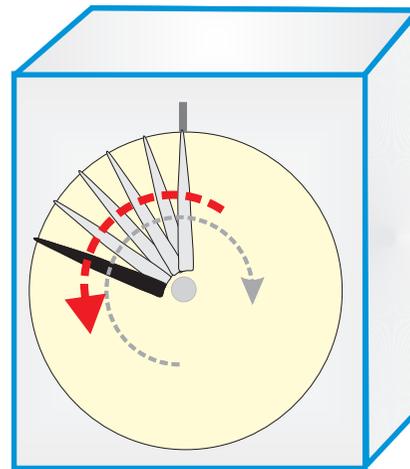
IF WE ASSUME CLOCK2 STARTS AT ZERO RANGE AND MOVES AWAY FROM CLOCK1, AT CONSTANT SPEED V , THE SERIES OF OVERLAID SNAP SHOTS THAT RESULTS IS SHOWN BELOW.

TO THE OBSERVER AT REFERENCE PLANE1 IT WOULD APPEAR THAT CLOCK2 IS RUNNING SLOW AS MEASURED AGAINST CLOCK1 AS THEY WOULD SEE CLOCK2 DIAL PRECESSING WITH EACH NEW SNAP SHOT TAKEN BY CLOCK1.

A CONSTANT DELTA CHANGE IN PHASE OF ONE CLOCK AGAINST ANOTHER INDICATES A CONSTANT RATE DIFFERENCE BETWEEN THE TWO CLOCKS.

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OBSERVER AT REFERENCE PLANE1 WOULD OBSERVE PRECESSION OF CLOCK2 DIAL WRT TO CLOCK1 DIAL AS SHOWN BY RED ARROW. THE RATE OF THIS PRECESSION IS DIRECTLY PROPORTIONAL TO SPEED V



OVERLAY OF DELAYED IMAGES OF CLOCK2. CLOCK2 IS MOVING AWAY FROM CLOCK1 AT SPEED V wrt REFERENCE PLANE1.

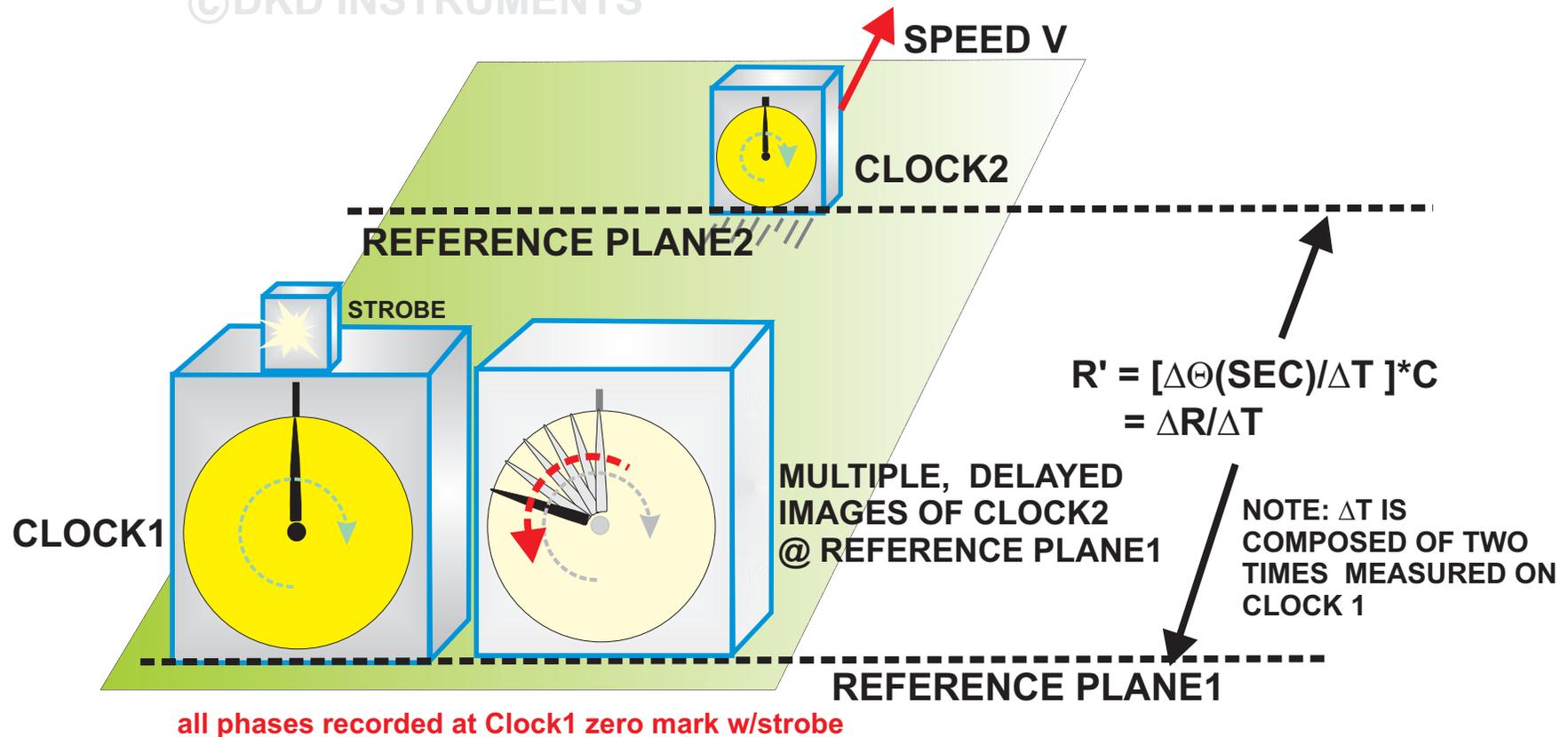
MEASURING THE SPEED OF A MOVING CLOCK

WE CAN NOW MEASURE V aka RANGE RATE, R' , BY USING PHASE MEASUREMENTS OF THE DELAYED IMAGE OF CLOCK2 RELATIVE TO CLOCK1.

AS NOTED THE RATE OF CLOCK2 IS PERCEIVED BY AN OBSERVER AT PLANE1 TO BE RUNNING SLOW FOR A RECEDING CLOCK2. IT WOULD BE PERCEIVED AS RUNNING FAST FOR AN APPROACHING CLOCK2.

FOR THE OBSERVER MOVING WITH CLOCK2 THE RATE WOULD BE UNCHANGED, IGNORING RELATIVISTIC EFFECTS. HENCE THE OBSERVED RATE DIFFERENCE OF CLOCK2@PLANE1 WRT TO CLOCK1 IS JUST A CONTINUOUS CHANGE OF PHASE DO THE CONTINUOUS CHANGE OF DELAY DUE TO SPEED V .

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MEASURING RANGE RATE, A WORKED EXAMPLE

BELOW WE SEE TWO IMAGES OF CLOCK2 TAKEN AT REFERENCE PLANE1 AND SEPARATED IN TIME BY 1 SECOND AS MEASURED BY CLOCK1.

TO BE CLEAR IMAGE1 IS TAKEN WHEN CLOCK1 DIAL HITS ZERO. IMAGE TWO IS TAKEN AFTER IMAGE1 BY EXACTLY 10,000 CYCLES OF CLOCK1'S 10uSEC DIAL. THEREFOR ΔT EQUALS 1 SECOND AS MEASURED BY CLOCK1.

FROM BELOW WE SEE THAT $\Theta_1=850\text{nsec}$ AND $\Theta_2=1075\text{nsec}$, THEREFORE CLOCK2 IS MOVING AWAY FROM CLOCK1

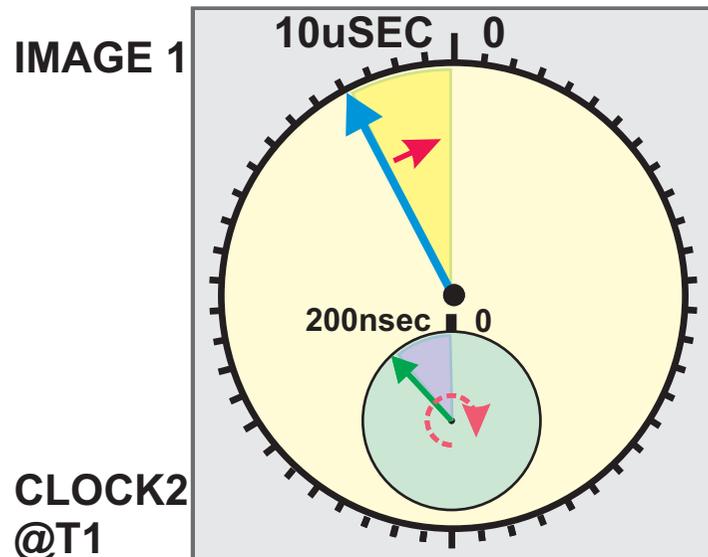
$$\Delta\Theta = \Theta_2 - \Theta_1; \text{ THEREFORE } \Delta\Theta = 225\text{nSEC}$$

NOW CONVERT TO DISTANCE, $\Delta R \sim 225 \text{ FEET } (C \sim 1\text{ft}/1\text{nsec})$

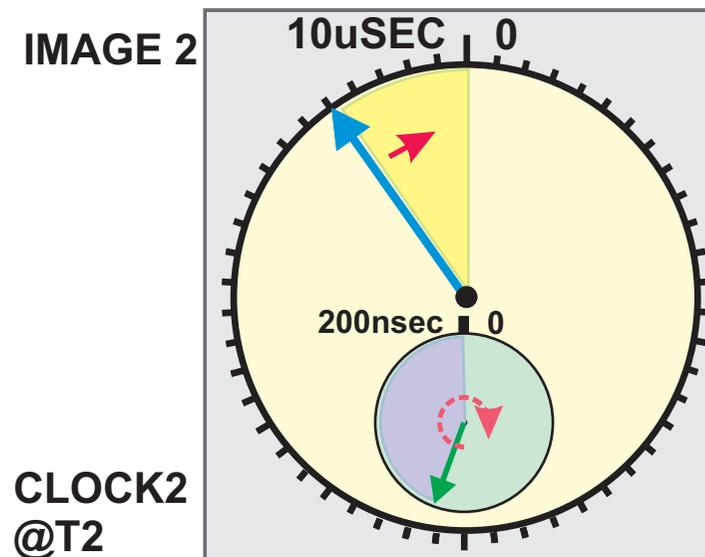
WITH $\Delta T = 1 \text{ SEC}$ THE RANGE RATE, $R' = 225\text{ft/SEC}$

all phases recorded at Clock1 zero mark

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$$\begin{aligned} \Theta_1(\text{sec}) &= 4 * 200\text{nsec} + 0.25 * 200\text{nsec} \\ &= 850\text{nsec} \end{aligned}$$



$$\begin{aligned} \Theta_2(\text{sec}) &= 5 * 200\text{nsec} + 0.375 * 200\text{nsec} \\ &= 1075\text{nsec} \end{aligned}$$

$$\Delta T = T_2 - T_1$$

THE RELATIONSHIP OF Θ TO CLOCK2 RANGE, SPEED AND ACCELERATION

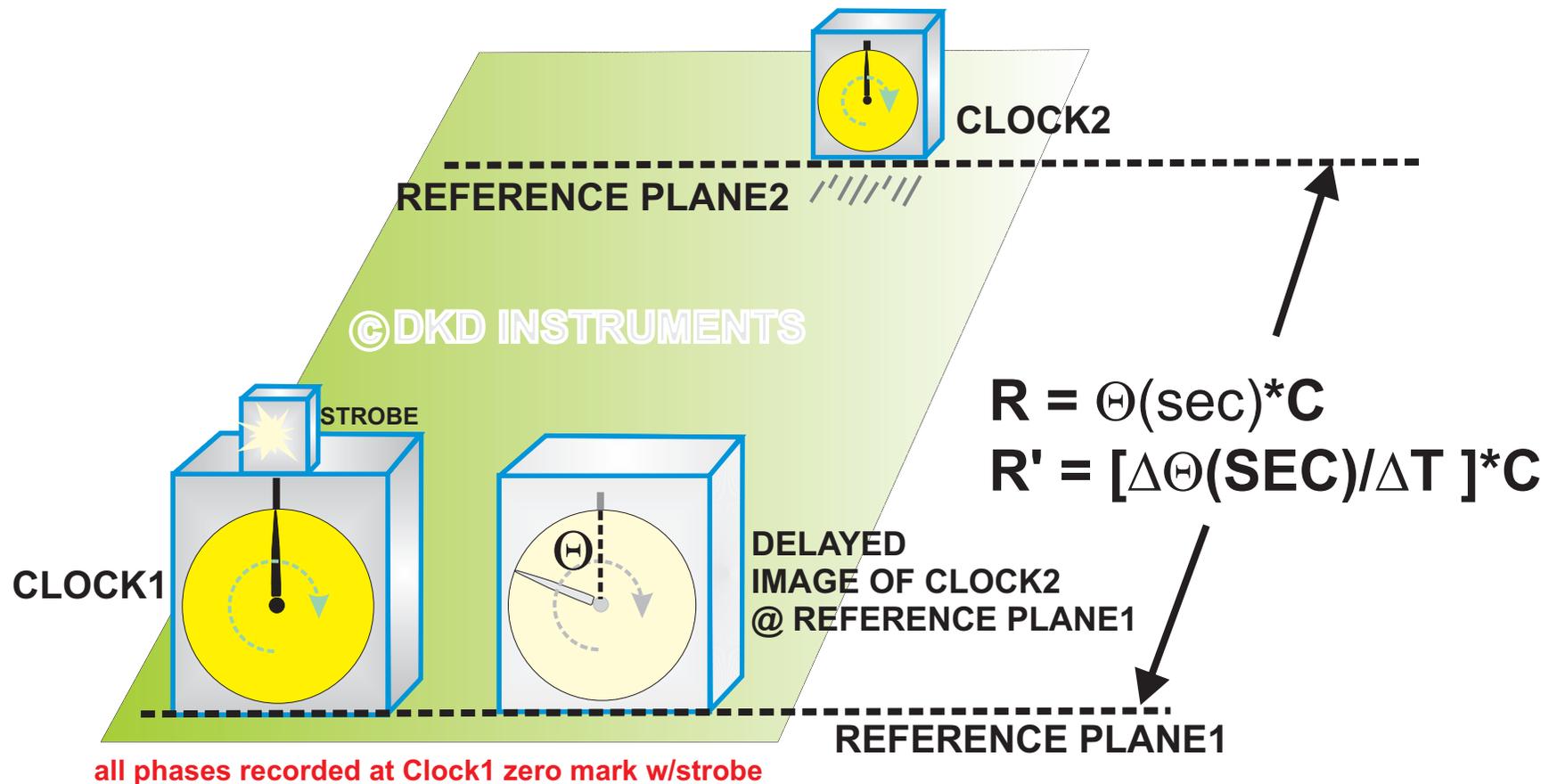
WE CAN NOW GENERALIZE THE RELATIONSHIP BETWEEN THE OBSERVED PHASE INFORMATION, Θ , OF CLOCK2 wrt TO REFERENCE PHASE PROVIDED BY CLOCK1;

$\Theta(t)$ IS DIRECTLY PROPORTIONAL TO RANGE BETWEEN CLOCK1 AND CLOCK2

$\Theta'(t)$ IS DIRECTLY PROPORTIONAL TO RANGE RATE OF CLOCK2

$\Theta''(t)$ IS DIRECTLY PROPORTIONAL TO ACCELERATION OF CLOCK2

WHERE $\Theta'(t)$ IS THE 1ST DERIVATIVE AND $\Theta''(t)$ IS THE SECOND DERIVATIVE.



FINDING THE APPARENT RATE DIFFERENCE OF CLOCK2 wrt CLOCK1

WE WISH TO DETERMINE THE RATE DIFFERENCE OF CLOCK2's IMAGE AS SEEN AT REFERENCE PLANE1 wrt CLOCK1 RATE. OUR TWO PHASE MEASUREMENTS RETURNED A RATE MEASUREMENT TO US OF;

$$\Delta\Theta = 225\text{nSEC IN 1 SECOND or } \Delta\Theta/\Delta T = 2.250 \text{ E-7 (NO UNITS!)}$$

THE ABOVE RATE HAS NO UNITS. IT IS ALSO A **RATE DIFFERENCE** wrt CLOCK1 RATE. (THIS IS BECAUSE ALL PHASES AND TIMES ARE USING CLOCK1 AS A REFERENCE)

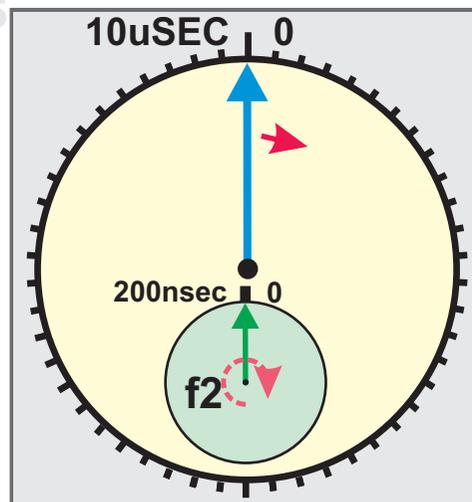
TO CONVERT THE ABOVE UNIT-LESS RATE INTO RATE TERM WITH UNITS OF CYCLES/SEC WE MUST MULTIPLY BY f2 RATE;

$$\Delta f = 5.0 \text{ E+6 (CYCLES/SEC) * } 2.250 \text{ E -7} \Rightarrow 1.125 \text{ CYCLES/SEC (Hz)}$$

THE APPARENT RATE OF f2' is; $5.0\text{MHz} - 1.125\text{Hz} = 4999998.875\text{Hz}$
(THE MINUS SIGN REFLECTS A RECEDING CLOCK2, WE WOULD ADD FOR AN APPROACHING CLOCK2)

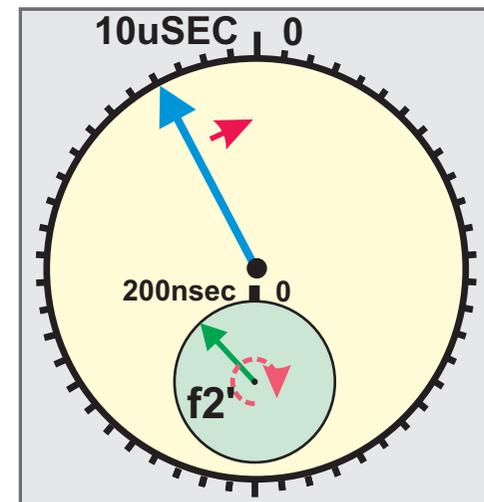
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CLOCK1



$$f_2 = 1/200\text{nsec} \Rightarrow 5\text{MHz}$$

IMAGE 1



CLOCK2
@T1

RATE EXAMPLE 2: RATE CALCULATIONS WITH INCREASED PRECISION

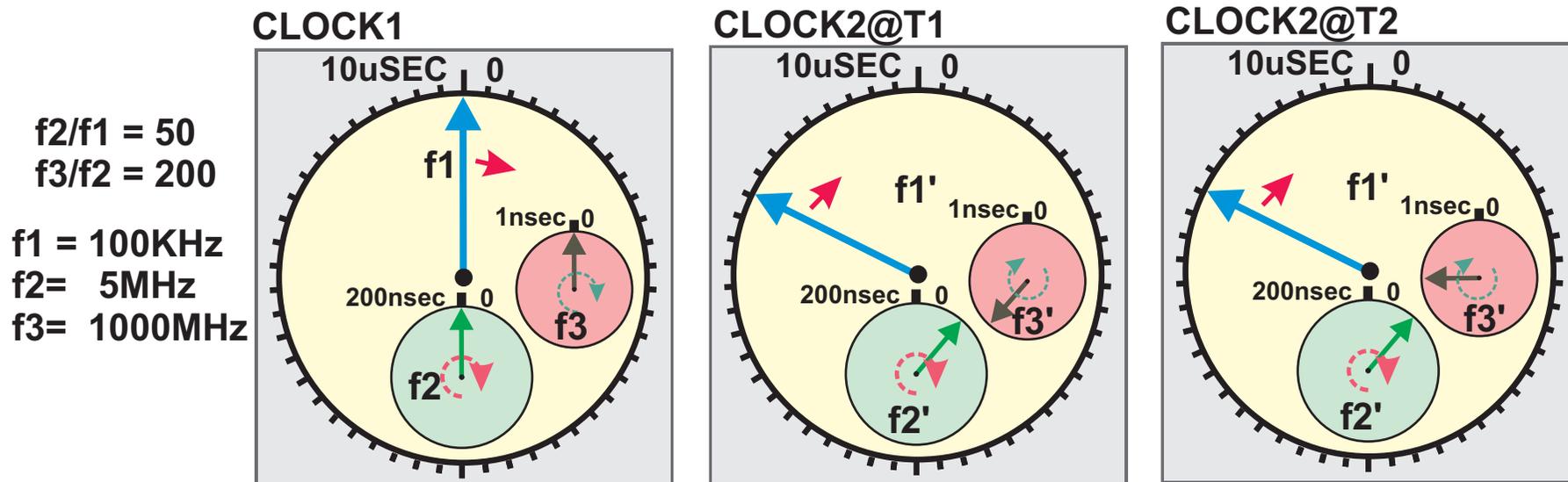
WE HAVE ADDED A NEW ANALOG DIAL THAT IS SYNCHRONOUS TO THE 200nsec DIAL TO BOTH CLOCK1 AND CLOCK2. THIS NEW DIAL COMPLETES A CYCLE IN 1nsec. OUR NEW DIAL IS VERY CLOSE TO THE L1 CARRIER DIAL IN RATE. FROM THE CLOCK2 IMAGES AT T1 AND T2 WE FORM Θ_1 AND Θ_2 ;

$$\left. \begin{aligned} \Theta_1 &= 9 \cdot 200\text{nsec} + 0.8125 \cdot 200\text{nsec} + 0.375 \cdot 1\text{nsec} \\ \Theta_2 &= 9 \cdot 200\text{nsec} + 0.8125 \cdot 200\text{nsec} + 0.25 \cdot 1\text{nsec} \end{aligned} \right\} \begin{aligned} \Delta\Theta &= -0.125\text{nsec} \\ \Delta\Theta/\Delta T &= -1.25\text{E-}10 \\ R' &\sim -0.125\text{ft/sec} \end{aligned}$$

FROM THE CALCULATIONS ABOVE WE SEE CLOCK2 IS APPROACHING CLOCK1 (VERY SLOWLY), THIS IS ALSO INDICATED BY THE NEGATIVE SIGN ON $\Delta\Theta$ AND R' . THE OBSERVED RATE OF f_3' AT REFERENCE PLANE1 IS;

$$\begin{aligned} f_3' &= 1000\text{MHz} - [-1.25\text{E-}10] \cdot 1000\text{MHz} \Rightarrow 1,000,000,000.125\text{Hz} \\ \text{USING RATIO'S; } f_2' &= f_3'/200 \Rightarrow 5,000,000.000625\text{Hz} \\ f_3' &= f_2'/50 \Rightarrow 100,000.0000125\text{Hz} \end{aligned}$$

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$f_2/f_1 = 50$
 $f_3/f_2 = 200$
 $f_1 = 100\text{KHz}$
 $f_2 = 5\text{MHz}$
 $f_3 = 1000\text{MHz}$

all phases recorded at Clock1 zero mark w/strobe

LITTLE DIALS, BIG RESULTS AND A DOPPLER CHECK

IT SHOULD BE CLEAR NOW THAT WHEN IT COMES TO RESOLUTION & PRECISION IN RANGE OR RANGE RATE MEASUREMENT, IT IS THE SMALLEST ,OR HIGH RATE DIALS OF OUR CLOCKS THAT DO THE HEAVY LIFTING ON THE PROBLEM.

WITH OUR ADDED 1nsec DIAL OF OUR LAST EXAMPLE WE MEASURED A RATE ERROR OF APPROXIMATELY 1 PART IN 10 BILLION! USING THE L1 CARRIER PHASE DIAL RUNNING AT A SLIGHTLY HIGHER RATE THAN OUR f3 DIAL AND IDEAL RECEIVER CONDITIONS, CLOCK RATE MEASUREMENT CAN APPROACH OR EXCEED 1 PART IN 100 BILLION. RESOLVING SUCH SMALL RATE ERRORS ARE THE PROVINCE OF ATOMIC CLOCKS. SUCH CLOCKS ARE USED EXTENSIVELY IN THE GPS SYSTEM.

RETURNING TO OUR LAST EXAMPLE WE CAN CHECK OUR CALCULATION OF OBSERVED RATE OF CLOCK2 AT PLANE1 BY USING STANDARD DOPPLER METHODS:

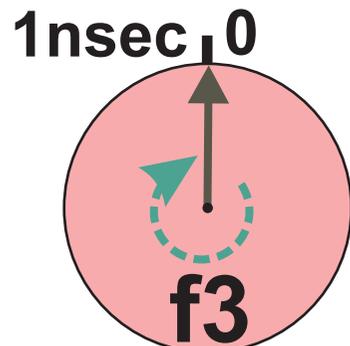
Observed frequency , $f_o = (1 - V/C) * f_s$

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where V is velocity, C is speed of light, f_s is frequency of source

for previous example, $V = - 1.25E-10 * C$, $f_s = 1000\text{MHZ}$

$f_o = (1 + 1.25E-10 * C/C) 1000\text{MHz} \Rightarrow 1,000,000,000.125 \text{ Hz}$, checks.

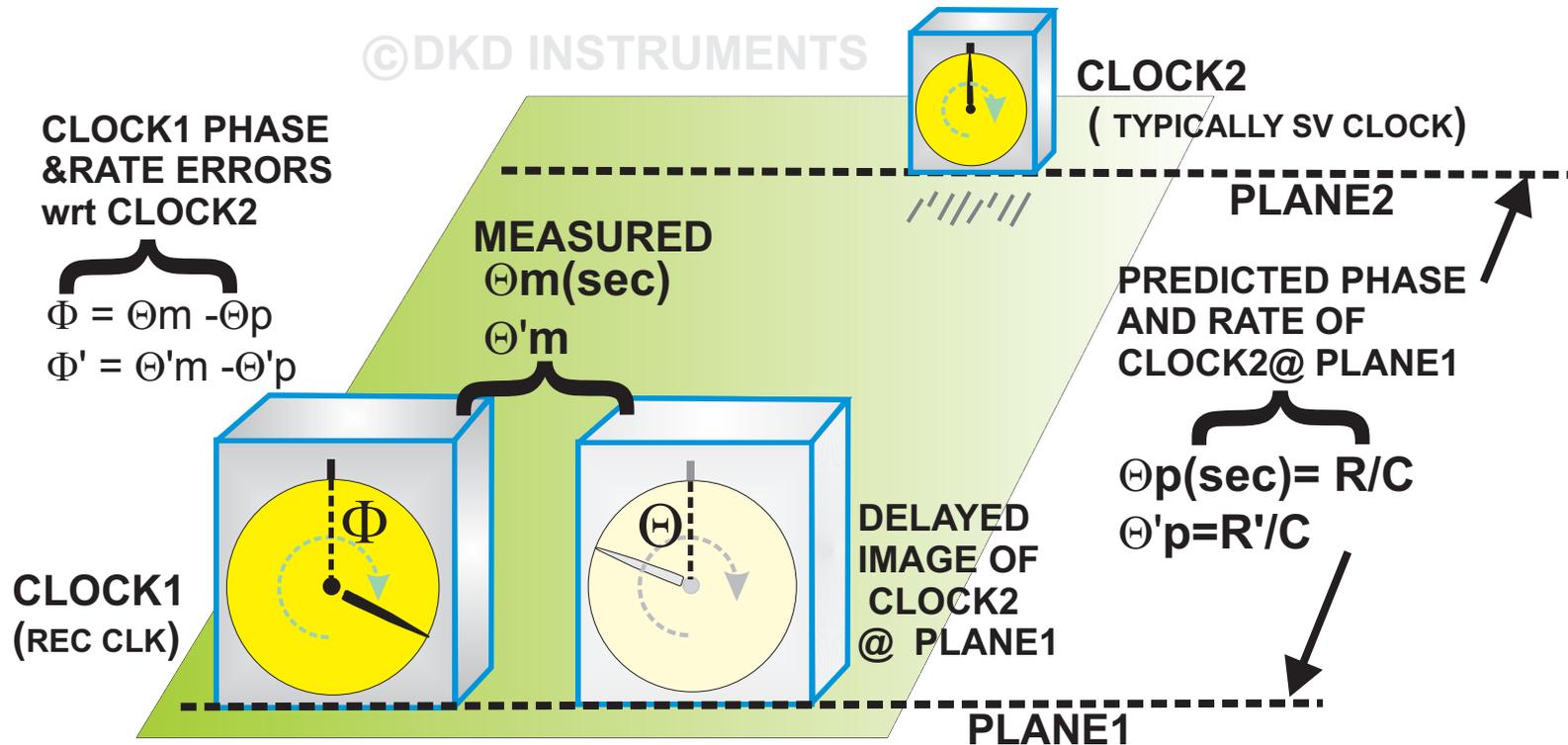


FINDING THE PHASE AND RATE ERRORS OF A LOCAL CLOCK

WE HAVE ASSUMED IN OUR TWO CLOCK SYSTEM THAT BOTH CLOCKS REMAINED IN SYNC. WITH A KNOWN RANGE AND RANGE RATE WE CAN USE THE SAME MODEL TO PREDICT RANGE AND RATE ERRORS OF EITHER CLOCK wrt TO THE OTHER CLOCK.

AS SEEN BELOW CLOCK2 IS ASSUMED TO BE THE MORE ACCURATE CLOCK AND WE WISH TO FIND OUT THE PHASE AND RATE ERRORS OF CLOCK1 wrt CLOCK2. WITH THE KNOWN R AND R' WE COMPUTE THE PHASE AND RATES OF CLOCK2 AS THEY WOULD BE OBSERVED AT PLANE1(PREDICTED). WE ALSO MEASURE (JUST AS BEFORE) THE PHASE AND RATE DIFFERENCES OF CLOCK2(@PLANE1) wrt CLOCK1. MEASURED minus PREDICTED IS THE PHASE AND RATE ERRORS OF CLOCK1 wrt CLOCK2.

THIS IS THE BASIS OF *GPS TIME TRANSFER*. CLOCK2 WOULD BE IN THE SATELLITE AND IS A VERY HIGH QUALITY ATOMIC CLOCK. CLOCK1 IS IN THE RECEIVER, TYPICALLY A LOW COST CLOCK. WITH A KNOWN (OR ESTIMATED) RECEIVER EARTH POSITION AND KNOWN SATELLITE ORBIT/POSITION INFORMATION THE RANGE AND RANGE RATE CAN BE PREDICTED.



all phases recorded at Clock2 zero mark

EXAMPLE OF DETERMINING A FIXED PHASE OFFSET OF CLOCK1

WE WILL ASSUME THAT THERE IS NO RATE ERROR ON CLOCK1 wrt TO CLOCK2 AND THAT CLOCK2 IS NOT MOVING. THE DISTANCE TO CLOCK2 FROM PLANE1 IS KNOWN TO BE ~ 2062ft. THE MEASURED PHASE DIFFERENCE BETWEEN CLOCK1 AND CLOCK2 AT PLANE1 IS .

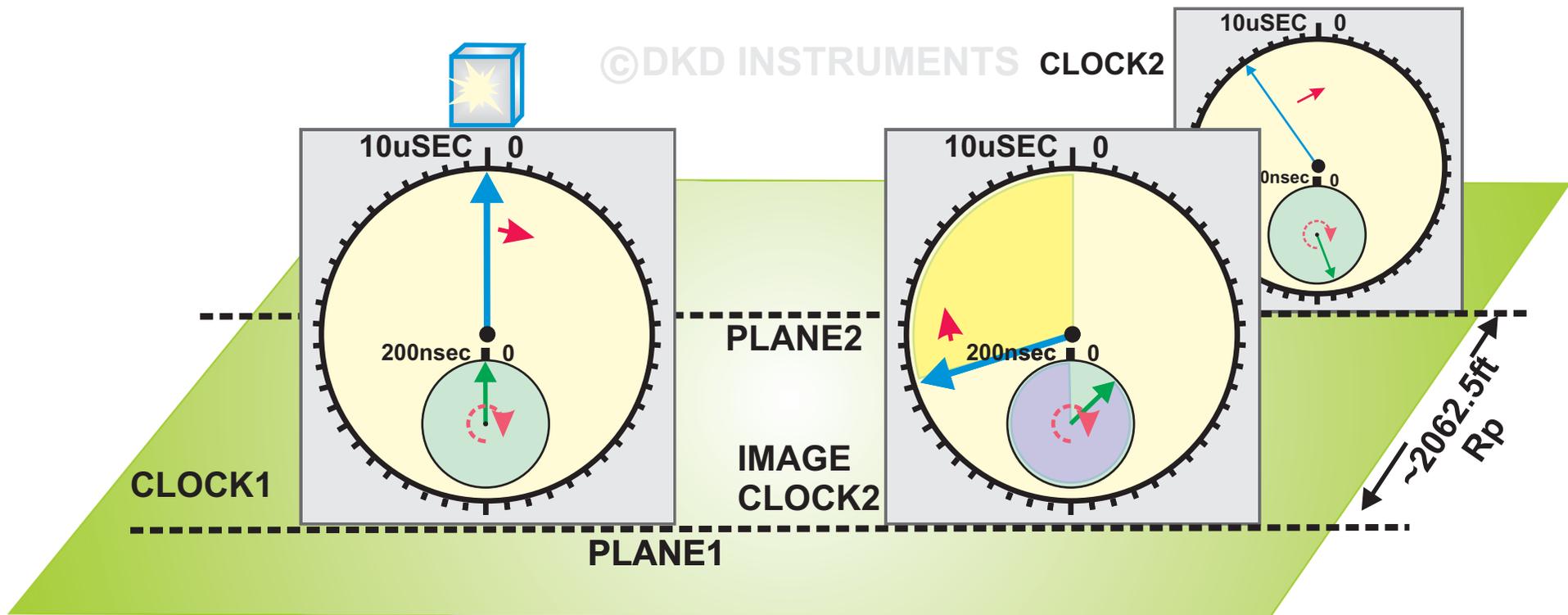
$$\Theta_m = 15 * 200\text{nsec} + 0.75 * 200\text{nsec} \Rightarrow 3175\text{nsec}, \text{ or MEASURED RANGE, } R_m \sim 3175\text{ft}$$

OUR MEASURED RANGE HAS AN ERROR DUE TO CLOCK PHASE ERROR , THUS ITS A *PSEUDO RANGE*.

THE PREDICTED PHASE IS $\Theta_p = 2062.5\text{ft} * 1\text{nsec/ft} \Rightarrow 2062.5\text{nsec}$

$$\Phi = \Theta_m - \Theta_p \Rightarrow 1112.5\text{nsec}$$

WE COULD JUST CHEAT AND READ THE PHASE ERROR OFF CLOCK2 DIAL AT PLANE2 wrt CLOCK1 @ PLANE1, *BUT IF CLOCK2 IS ON A SATELLITE YOU CANT "SEE" IT.*



all phases recorded at Clock1 zero mark w/strobe

A MISSING DIAL BETWEEN C/A CHIP AND CARRIER PHASE DIALS?

THE PHASE RELATIONSHIP BETWEEN THE L1 GPS CLOCK C/A CHIP DIAL AND THE CARRIER PHASE DIAL COULD USE SOME HELP. THE RATIO BETWEEN THESE TWO DIALS IS 1540:1. TYPICALLY ADJACENT DIAL RATIO'S SHOULD BE BETWEEN 10 AND 100. THE C/A CHIP DIAL TO C/A CODE DIAL ALSO BREAKS THE 10 TO 100 RULE. THOSE TWO DIALS HAVE A SPECIAL RELATIONSHIP THAT ALLOWS SUCH A LARGE RATIO(1023:1). THE C/A CHIP AND CARRIER PHASE DIALS COULD REALLY USE A DIAL MIDWAY BETWEEN THEM. IN TERMS OF RATIO, A $\sim 40:1$ WOULD HAVE BEEN IDEAL. (RATE $\sim 40\text{MHz}$)

THE ABSENCE OF THIS DIAL LEADS TO A PROBLEM OF PHASE UNCERTAINTY BETWEEN THE C/A CHIP AND CARRIER PHASE DIALS. SINCE LOCKING THEM IN PHASE IS USUALLY NOT POSSIBLE THEY ARE TIED ONLY IN RATE OR SYNTONY. IN PARTICULAR THE CARRIER PHASE DIAL RATE IS ALLOWED (IN SOME RECEIVERS) TO DRIVE THE C/A CHIP DIAL RATE, OR SO CALLED **CARRIER AIDED TRACKING**.

THE PHASE ISSUE BETWEEN THESE TWO DIALS IS THE FUNDAMENTAL ISSUE THAT DRIVES THE DISTINCTION BETWEEN *CODE PHASE* MEASUREMENTS AND *CARRIER PHASE* MEASUREMENTS.

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PART II
L1 CLOCK SIGNAL TRANSMISSION,
RECEPTION, SYNCHRONIZATION & RANGE MEASUREMENT
WITH
MECHANICAL ROTARY CLOCK MODELS

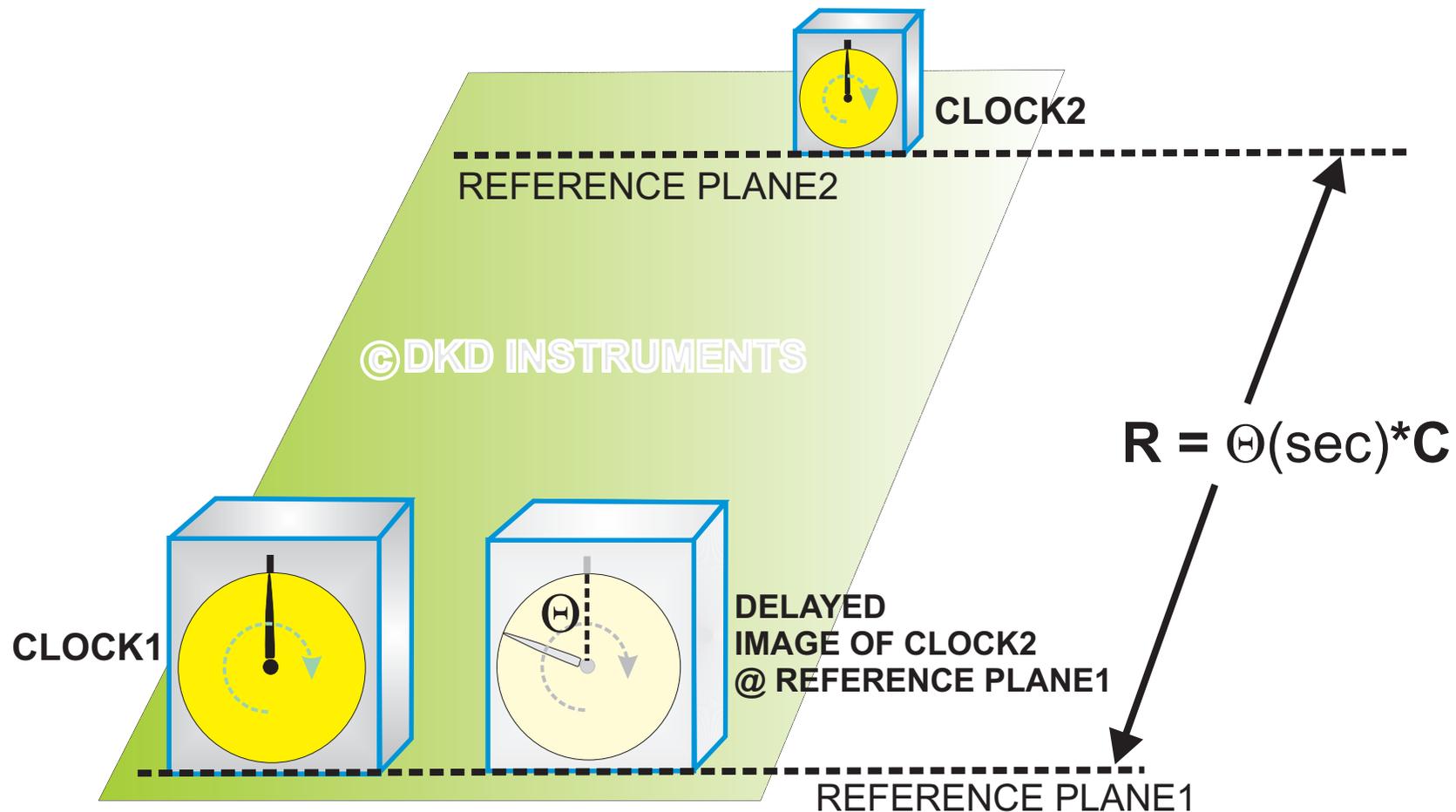


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PREVIOUSLY WE SHOWED HOW WE CAN USE THE IMAGE OF A DISTANT CLOCK (CLOCK2) TO MEASURE RANGE BY PHASE COMPARISON WITH A LOCAL CLOCK THAT IS IN SYNC WITH IT , aka CLOCK1.

WHILE COMPLETELY FUNCTIONAL FROM AN ANALYSIS POINT OF VIEW ITS OBVIOUSLY NOT PRACTICAL.

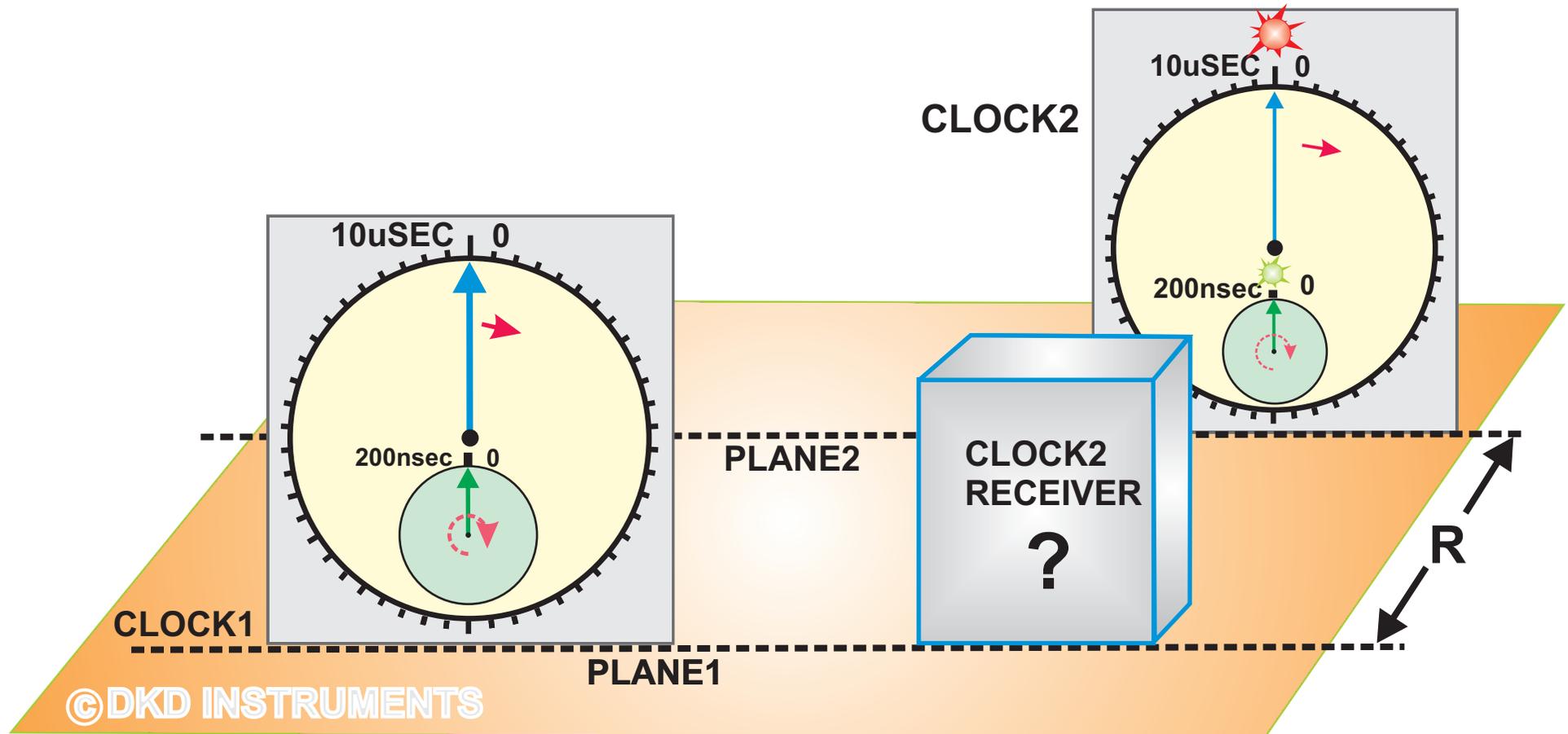
WE NEED A WAY TO SEND *CLOCK SIGNALS* FROM THE DISTANT CLOCK TO WHERE OUR LOCAL CLOCK IS. GPS USES RADIO WAVES TO ACCOMPLISH THIS TASK. THE PROBLEM WITH RADIO WAVES IS WE CANT SEE THEM.



A CLOCK SIGNAL TRANSMITTER USING LIGHT FLASHES

BELOW WE HAVE SETUP OUR NEW MODEL FOR SENDING AND RECEIVING CLOCK INFORMATION FROM CLOCK2 @ PLANE2. CLOCK1 AND CLOCK2 ARE ASSUMED TO BE IN SYNC. CLOCK1 AND CLOCK2 ARE BOTH COMPOSED OF A 10uSEC DIGITAL DIAL WITH 50 TIC'S AND A 200nsec ANALOG DIAL.

CLOCK2 HAS TWO LASER LIGHTS THAT FLASH WHEN EVER THEIR DIALS PASS THE ZERO TIMING MARK. THE 10uSEC LIGHT FLASHES RED, THE 200nsec LIGHT FLASHES GREEN. THE RECEIVER WILL SEE THESE LIGHT FLASHES DELAYED BY THE RANGE R. ITS THE RECEIVERS JOB TO TAKE THE LIGHT FLASHES AND RECONSTRUCT CLOCK2 AS IT WOULD APPEAR AT PLANE1. WHAT IS INSIDE CLOCK2 RECEIVER?



LIGHT PULSE RECEIVER OF CLOCK SIGNALS

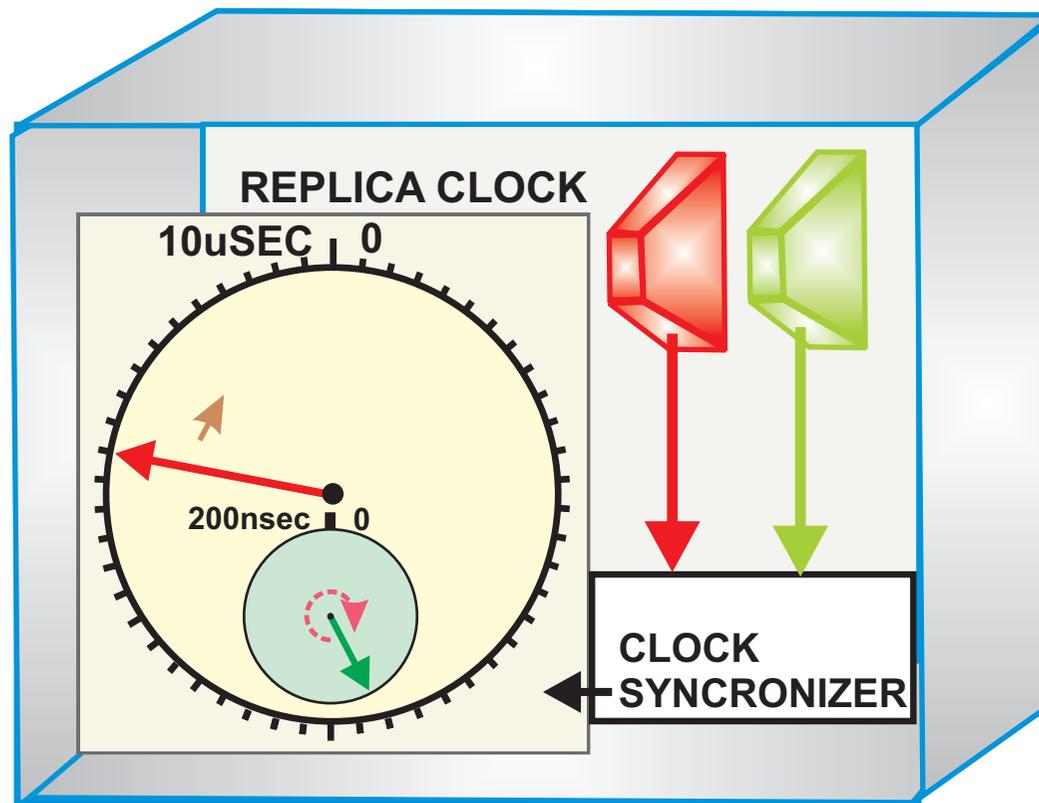
OUR RECEIVER IS COMPOSED OF THREE FUNCTIONS: RED & GREEN LIGHT SENSORS, A CLOCK SYNCHRONIZER AND A REPLICA OF CLOCK1,2.

IN OPERATION THE REPLICA CLOCK IS STARTED AT THE ESTIMATED RATE OF CLOCK2's 200nsec DIAL . THE PHASE AND RATE OF THE REPLICA CLOCK ARE THEN ADJUSTED SO THAT THEY AGREE WITH THE RECEIVED RED AND GREEN LIGHT PULSES SENT FROM CLOCK2.

THE JOB OF ADJUSTING THE REPLICA CLOCK PHASE AND RATE IS DONE BY THE CLOCK SYNCHRONIZER. THE CLOCK SYNCHRONIZER IS THE TRUE HEART OF THE RECEIVER.

IN FUNCTION THIS MODEL IS VERY CLOSE TO WHAT HAPPENS INSIDE A GPS RECEIVER. FOR EACH SATELLITE TRACKED OR RECEIVED THERE IS A REPLICA CLOCK AND A CLOCK SYNCHRONIZER FOR IT.

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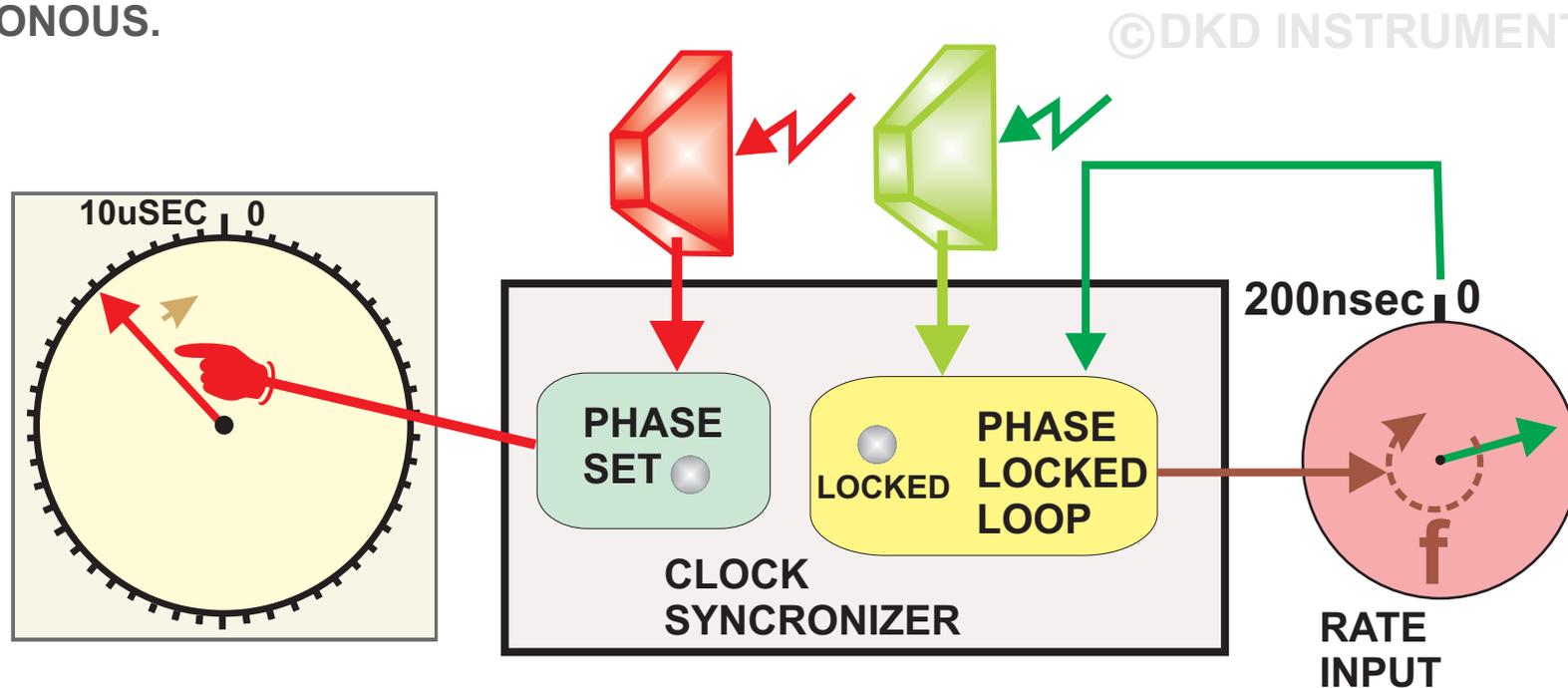


CLOCK SYNCHRONIZER FUNCTIONS

THE CLOCK SYNCHRONIZER USES THE RECEIVED RED AND GREEN LIGHT PULSES IN DIFFERENT WAYS. OUR TWO DIAL CLOCK IS SPLIT TO SHOW THIS BUT THE DIALS ARE STILL CONNECTED AS BEFORE.

THE GREEN PULSE IS USED TO PHASE LOCK THE 200nsec DIAL TO CLOCK2's 200nsec DIAL. WHEN LOCKED THE REPLICA CLOCK 200nsec DIAL WILL HAVE THE SAME RATE AND PHASE OF A DELAYED CLOCK2 IMAGE AT PLANE1 (AND THE LOCK LITE WILL BE LIT). THE PHASE LOCK BOX SENSES THE PHASE DIFFERENCE BETWEEN THE PHASE OF IT'S 200nsec DIAL AND THE INCOMING GREEN LIGHT PULSE (or PHASE). IT KEEPS ADJUSTING THE RATE OF THE 200nsec DIAL TO KEEP IT LOCKED TO THE RECEIVED GREEN CLOCK SIGNAL.

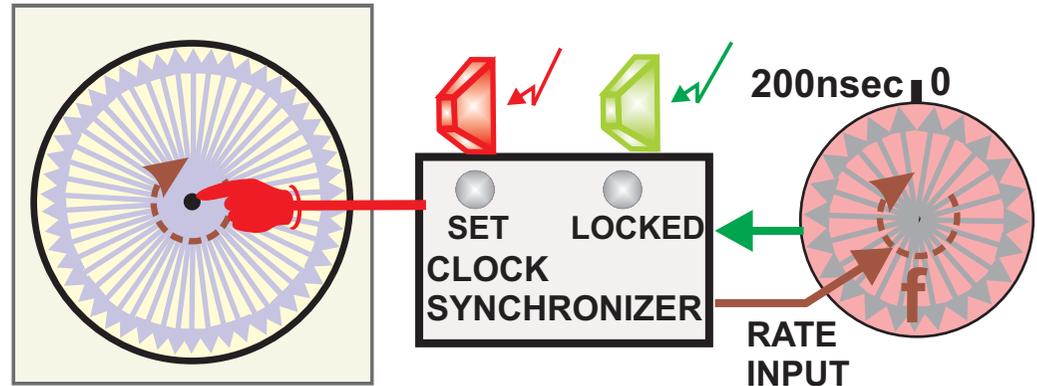
THE RED LIGHT PULSE IS USED ONLY WHEN THE PHASE OF THE REPLICA CLOCKS 10usec DIAL NEEDS TO BE RESET TO MATCH THE INCOMING RED LIGHT PULSE(OR PHASE). ONCE ITS PHASE IS SET (SET LITE ON) IT WILL NOT DRIFT IN PHASE AS LONG AS THE 200nsec DIAL IS HELD IN PHASE LOCK. ADDITIONALLY THE RATE OF THE 10usec DIAL IS ESTABLISHED BY THE RATE OF THE 200nsec DIAL AS THESE DIALS ARE SYNCHRONOUS.



FROM UNLOCKED TO LOCKED, aka ACQUISITION

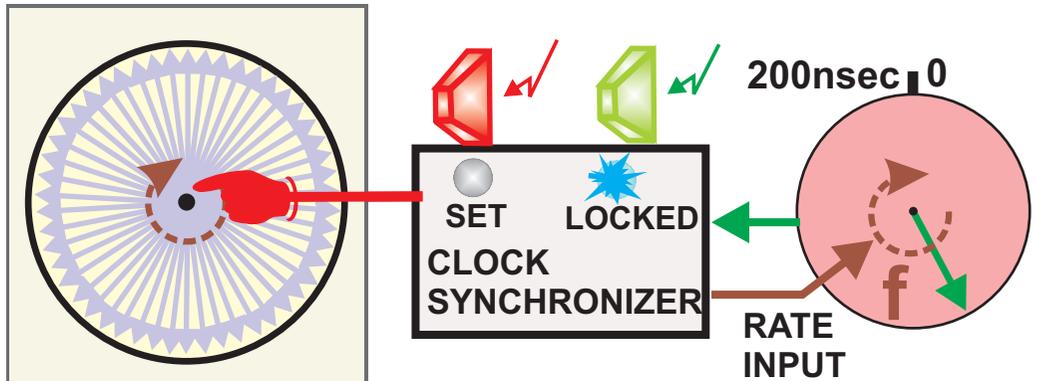
THE PROCESS OF THE RECEIVER REPLICA CLOCK GOING FROM UNLOCKED TO LOCKED DOES NOT HAPPEN ALL AT ONCE. IT IS A SEQUENCE OF STAGES THAT OCCURS

WITH NO PRIOR PHASE KNOWLEDGE WE SET 200nsec DIAL TO NOMINAL RATE OF 5MHz. IF CLOCK2 IS MOVING THERE WILL BE A RATE DELTA. THE PHASE OF BOTH DIALS IS UNKNOWN. THE 200nsec DIAL PHASE CAN BE ANYTHING AS ITS AN ANALOG DIAL. THE 10usec DIAL PHASE AND TIMING MARKS ARE UNKNOWN.

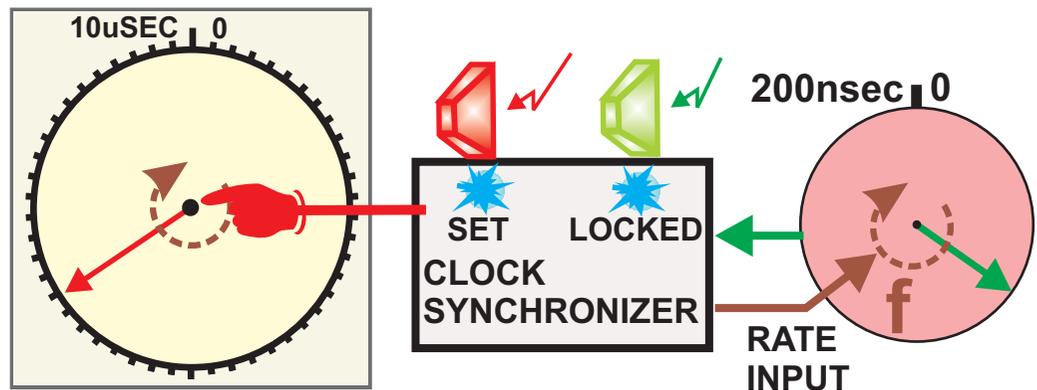


THE 200nsec DIAL IS NOW LOCKED AND THE LOCKED LITE IS ON. THE PHASE AND RATE OF THIS DIAL NOW MATCHES CLOCK2 AS IT WOULD BE OBSERVED AT PLANE1.

THE 10uSEC DIAL PHASE IS STILL NOT CORRECT BUT ITS RATE IS, THIS IS DUE TO LOCK OF 200nsec DIAL.



THE PHASE OF THE 10usec DIAL IS SET TO THE RECEIVED RED LITE PULSE, THE SET LITE IS ON. THE PHASE AND RATE OF REPLICA CLOCK NOW REPRESENTS CLOCK2 PHASE AND RATE AS SEEN AT PLANE1. THE CLOCK SYNCHRONIZER WILL CONTINUOUSLY ADJUST THE 200usec DIAL RATE TO MAINTAIN LOCK.

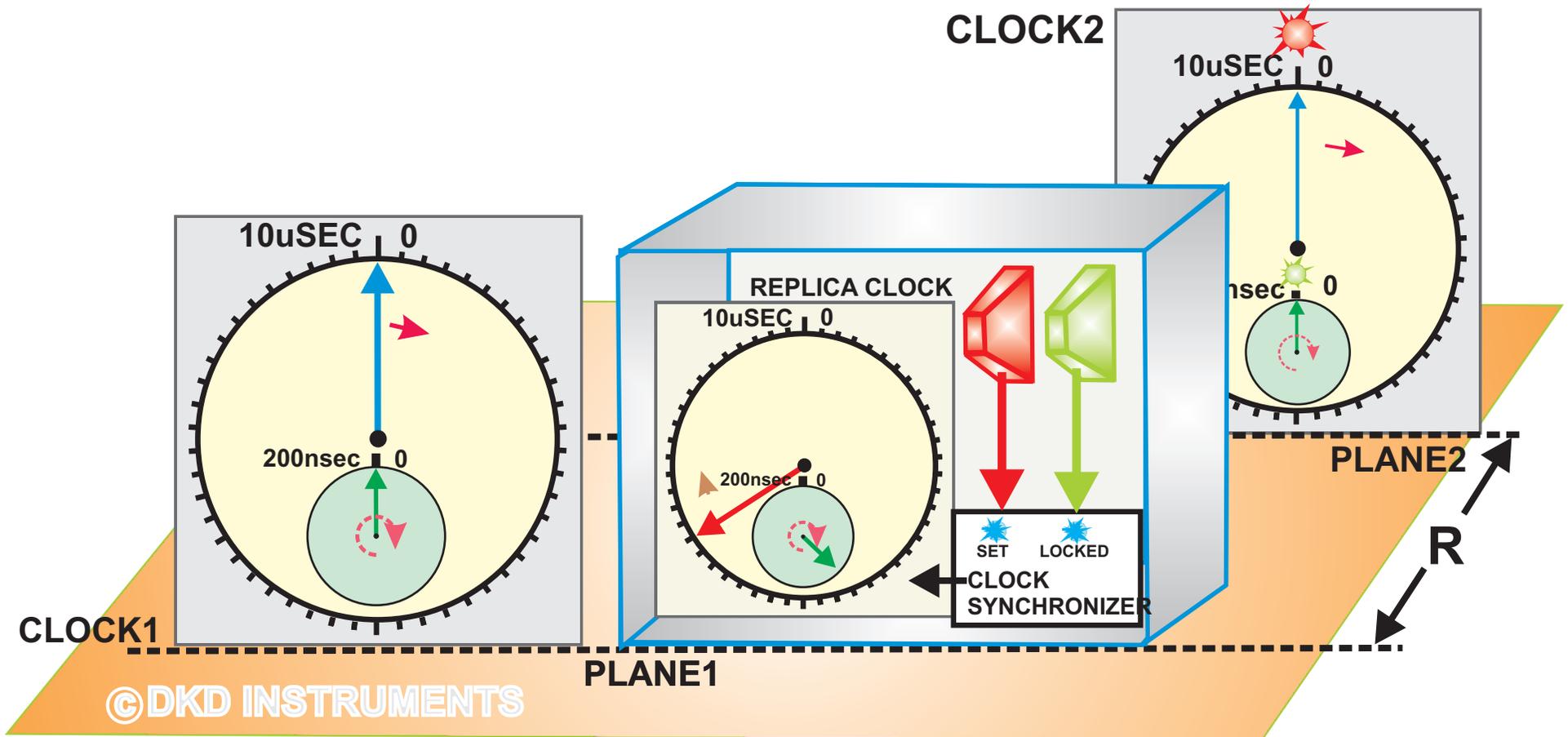


ALL PHASES TAKEN AT THE INSTANT CLOCK1 DIAL IS AT ZERO TIMING MARK

WITH ACQUISITION COMPLETE WE CAN NOW USE THE REPLICA CLOCK PHASE TO MEASURE THE RANGE R. AS BEFORE WE ASSUME NO PHASE OR RATE ERRORS BETWEEN CLOCK1 AND CLOCK2;

$$\Theta = 17 * 200\text{nsec} + 0.625 * 200\text{nsec} \Rightarrow 3525\text{nsec} \text{ , using } C \sim 1\text{nsec/ft} \text{ , } R \sim 3525\text{ft}$$

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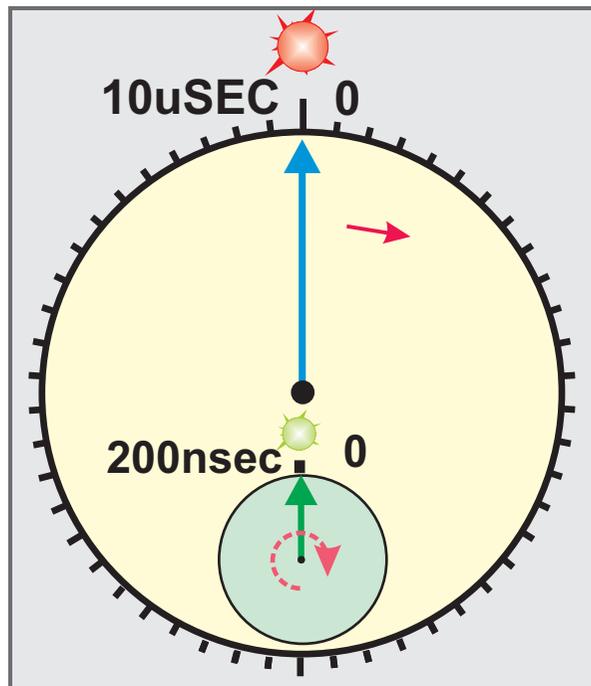
ANOTHER WAY TO SEND CLOCK2 TIMING SIGNALS

IF OUR SIMPLE CLOCK2 TRANSMITTER WE USED TWO LASER LIGHTS TO TRANSMIT CLOCK2 PHASE INFORMATION. IF WE WISH WE COULD USE JUST ONE, THE RED LIGHT OF THE 10uSEC DIAL. THIS CAN BE DONE BY SENDING A CODE INSTEAD OF SENDING JUST ONE BLINK PER CYCLE.

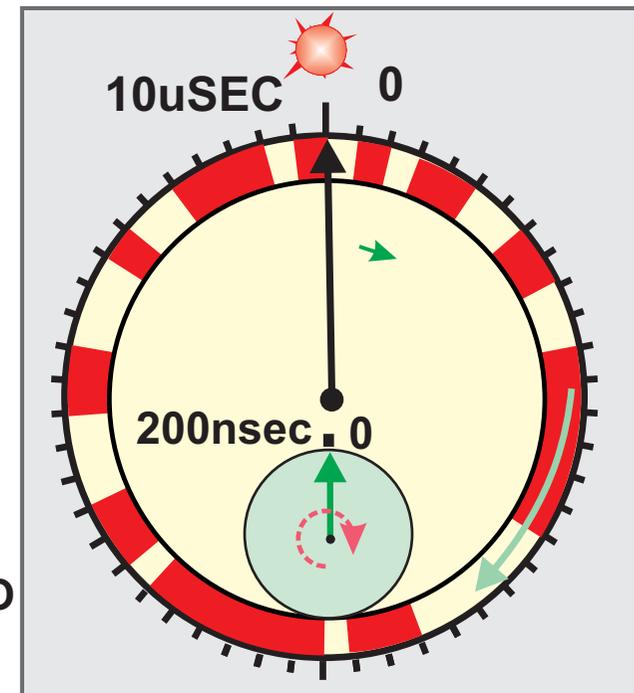
AS SHOWN BELOW THE DIAL POINTER NOW HAS WHEEL ATTACHED TO IT. WHEN A RED PORTION OF THE WHEEL IS AT THE ZERO TIMING MARK THE RED LIGHT IS ON. IF A WHITE SECTION IS AT THE ZERO TIMING MARK THE RED LIGHT IS OFF. THE TRANSITIONS FROM RED TO WHITE OR WHITE TO RED ONLY HAPPEN AT 200nsec INTERVALS, WHICH IS THE TIC SPACING ON THE 10uSEC DIAL.

THIS METHOD IS AN EXAMPLE OF ENCODING TO DIALS ONTO ONE SIGNAL BY USE OF A BINARY CODE. THE CODE USED IS CALLED PSEUDO RANDOM CODE.

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CLOCK2



CLOCK2 WITH PSEUDO
RANDOM CODED DIAL
TRANSMITTER

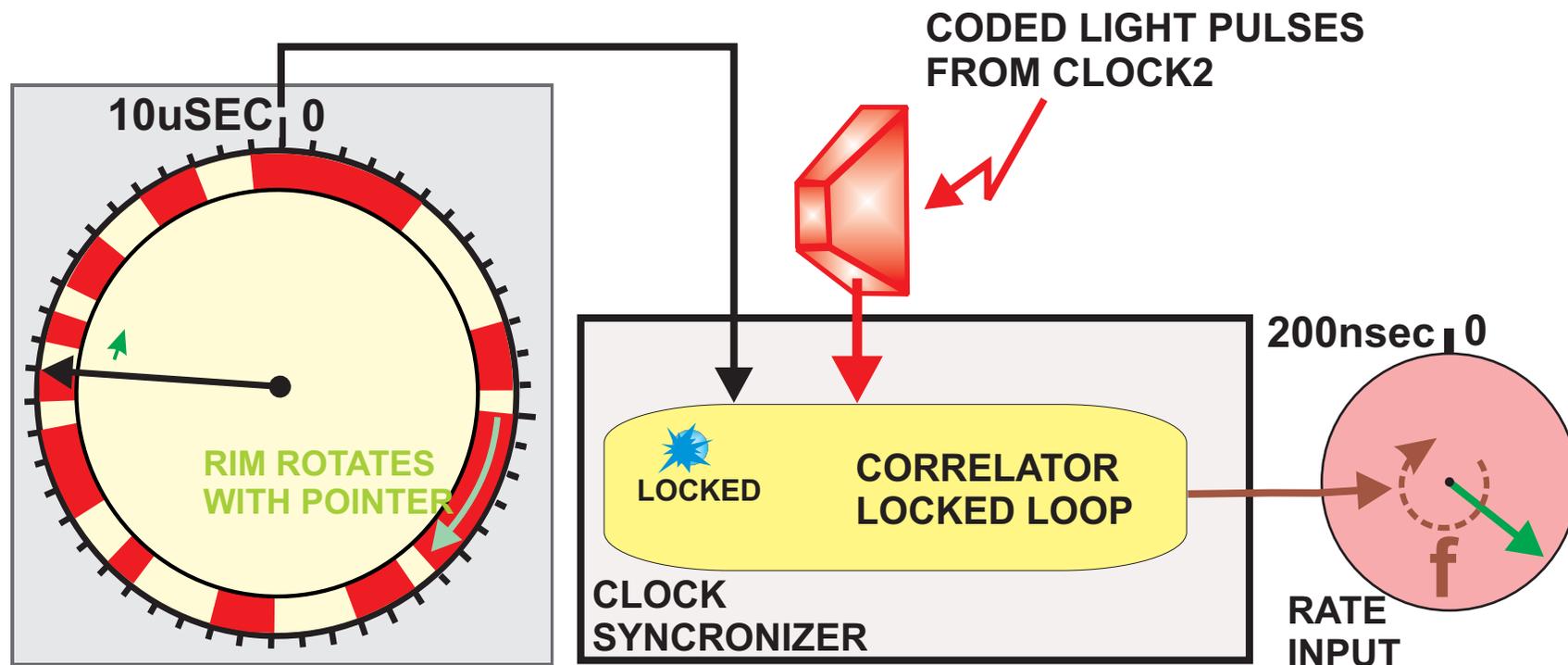
CORRELATION TYPE RECEIVER CLOCK SYNCHRONIZER

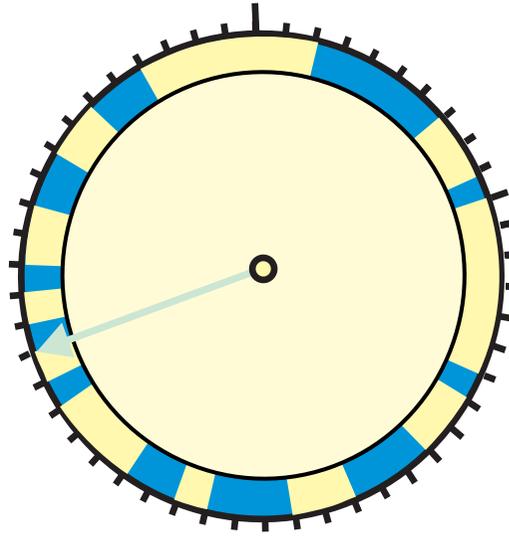
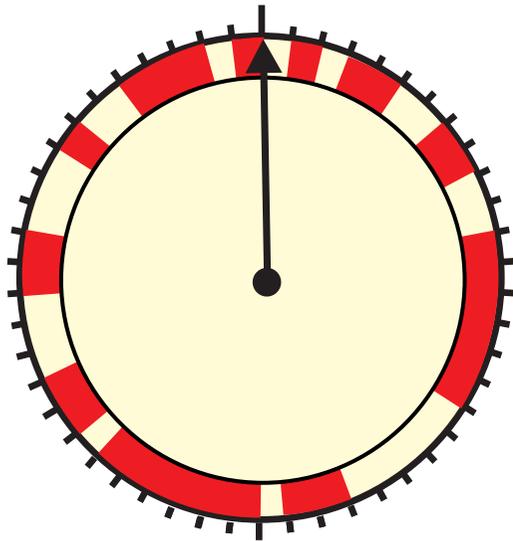
THE CHANGE IN THE WAY WE SEND THE CLOCK INFORMATION MEANS WE WILL HAVE TO CHANGE OUR RECEIVER. SPECIFICALLY THE REPLICA CLOCK MUST NOW ALSO HAVE THE SAME CODE ON ITS 10uSEC DIAL AS IS BEING TRANSMITTED.

THE CLOCK SYNCHRONIZER IS NOW OF THE *CORRELATOR* TYPE. THE SYNCHRONIZER COMPARES THE RECEIVED CODE WITH THAT COMING FROM THE REPLICA CLOCK 10uSEC DIAL. THE SYNCHRONIZER CHANGES THE RATE OF THE 200nsec DIAL IN ORDER TO ROTATE THE REPLICA CLOCK CODE DIAL WITH RESPECT TO RECEIVED CODE. WHEN THE CODES MATCH THE PHASE OF *BOTH* THE 200nsec AND 10uSEC DIALS HAS BEEN CORRECTLY RECEIVED(or RECOVERED) AND THE LOCK LIGHT IS TURNED ON. NOTE THAT THE PHASE OF 200nsec DIAL IS NOT PART OF THE SYNCHRONIZER OPERATION AS IT WAS IN TH PLL/ TWO LIGHT METHOD.

ONCE LOCK HAS OCCURRED WE CAN MEASURE THE RANGE TO CLOCK2 JUST AS BEFORE.

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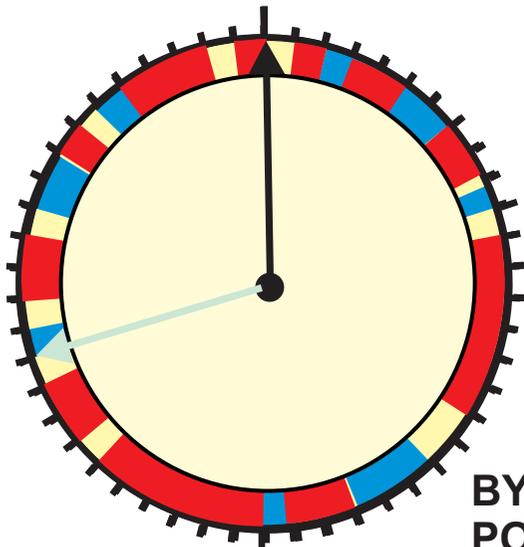




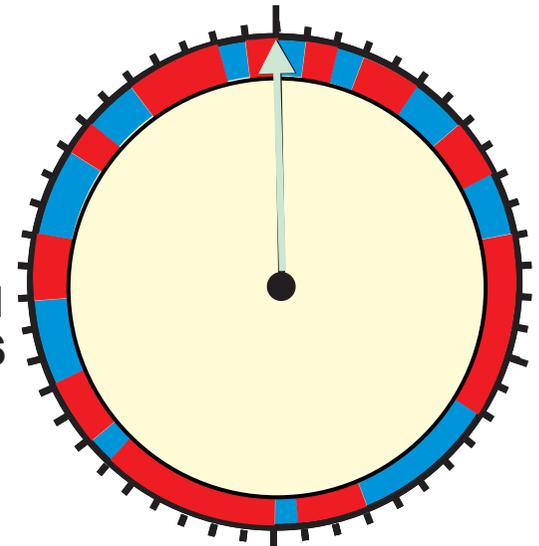
TO ILLUSTRATE THE CORRELATION PRINCIPLE WE START WITH TWO CODED 10usec DIALS , THE BLUE DIAL CODE IS INVERTED wrt TO RED DIAL.

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IF WE OVERLAY THE TWO DIALS WE SEE THAT THE POINTERS ARE NOT IN PHASE. WE CAN ALSO SEE LIGHT COLORED AREAS BETWEEN RED/BLUE SECTIONS.



BY ROTATING THE BLUE DIAL THE TWO POINTERS ARE IN PHASE AND THERE IS NO LIGHT COLORED SECTIONS BETWEEN THE RE/BLEU SECTIONS, THE TWO DIALS ARE NOW *CORRELATED*. CORRELATION ON ROTARY CLOCKS IS LIKE A CIRCULAR LOCK AND KEY. THE TWO WHEELS MUST BE WITHIN ONE CODE BIT , OR CHIP, FOR CORRELATION LOOP TO REMAIN LOCKED.



ENCODING A LOWER RATE DIAL ON A HIGHER RATE DIAL USING CW/CCW PHASE MODULATION

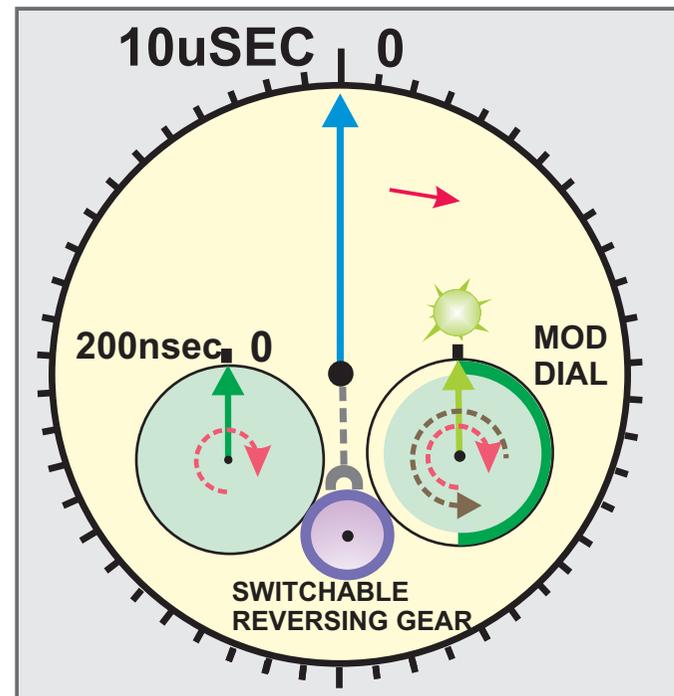
ANOTHER WAY WE CAN TRANSMIT TWO DIALS USING ONE LIGHT IS *PHASE MODULATION*. BELOW WE HAVE ADDED ANOTHER 200nsec ANALOG DIAL WHICH WE CALL THE *MODULATION DIAL*. THE EXISTING 200nsec DIAL DRIVES THE MODULATION DIAL THROUGH A REVERSIBLE 1:1 GEAR BOX. THE MODULATION DIAL HAS A BI-COLORED RING WHICH DETERMINES IF THE LIGHT IS LIT OR NOT. IF THE ZERO TIMING MARK IS OVER THE LIGHT COLORED PORTION OF THE RING THE GREEN LITE IS OFF. WHEN THE ZERO TIMING MARK IS OVER THE GREEN PORTION OF THE RING THE GREEN LIGHT IS ON.

THE PHASE INFORMATION OF THE 10usec DIAL IS ENCODED ONTO THE MODULATION DIAL BY *REVERSING THE DIRECTION OF ROTATION* OF MODULATION DIAL EVERY TIME THE 10usec DIAL CROSSES ITS ZERO TIMING MARK (A TYPE OF BI-PHASE MODULATION). THEREFORE EVERY 50 CYCLES THE MODULATION DIAL REVERSES DIRECTION. THE MODULATION DIAL NOW CONTAINS PHASE INFORMATION OF BOTH 10usec AND 200nsec DIALS.

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AN ALTERNATIVE WAY TO ENCODE 10usec DIAL PHASE ONTO MODULATION DIAL IS TO ADD 180DEGRESS OF PHASE FOR EVERY CYCLE OF THE 10usec DIAL.

**CLOCK2 TRANSMITTER
WITH CW/CCW PHASE
MODULATION DIAL**



PHASE MODULATION TYPE RECEIVER/CLOCK SYNCHRONIZER

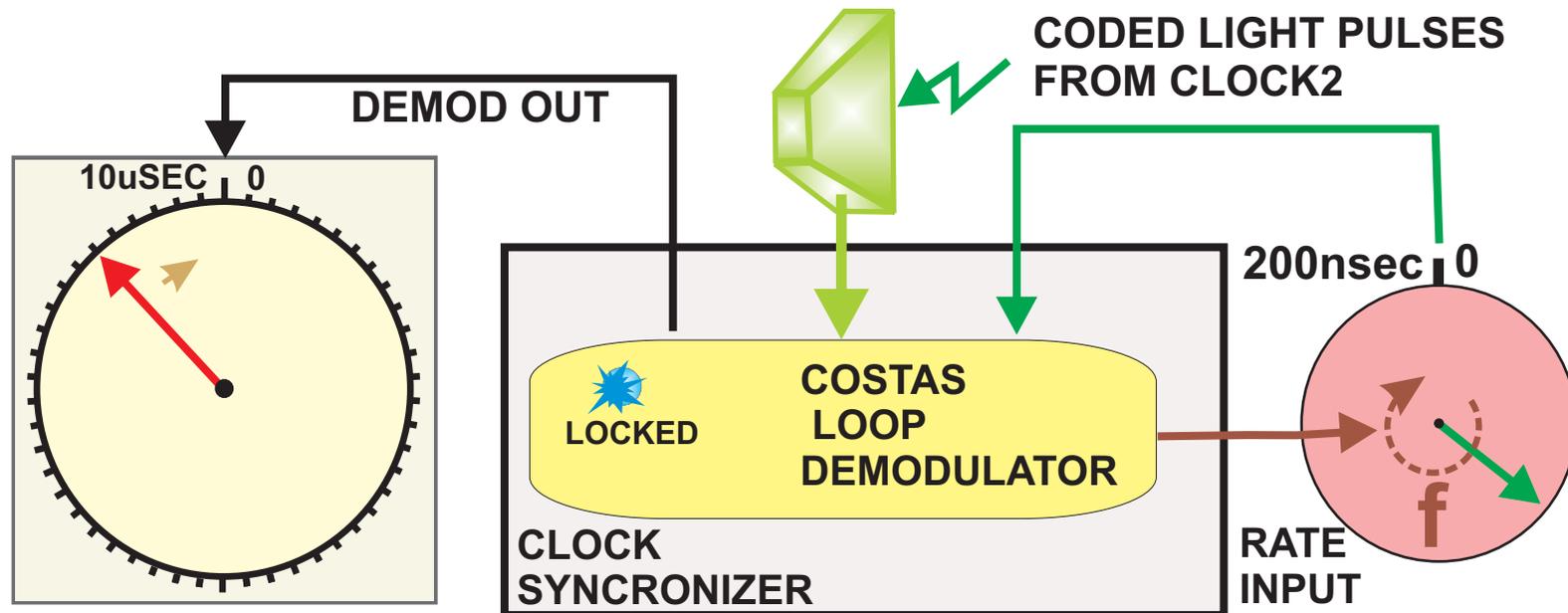
RECOVERY OF THE RECEIVERS REPLICA CLOCK FROM A BI-PHASE MODULATED CLOCK SIGNAL CAN BE DONE WITH A SPECIAL TYPE OF DEMODULATOR CALLED A *COSTAS LOOP DEMODULATOR*.

COSTAS LOOP DEMODULATORS UNDO THE PHASE MODULATION ON THE 200nsec DIAL TRANSMISSION AND RECOVER THE 200nsec DIAL PHASE. THE COSTAS LOOP IS A SPECIAL TYPE OF PHASE LOCKED LOOP USED RECOVER THE 200nsec DIAL.

WHEN THE LOOP IS LOCKED THE PHASE MODULATION IS REMOVED THUS RECOVERING THE 200nsec DIAL. THE MODULATION REMOVED FROM THE 200usec DIAL IS THE 10usec DIAL PHASE INFORMATION.

WITH THE LOOP LOCKED OUR REPLICA CLOCK IS READY TO MAKE RANGE MEASUREMENTS.

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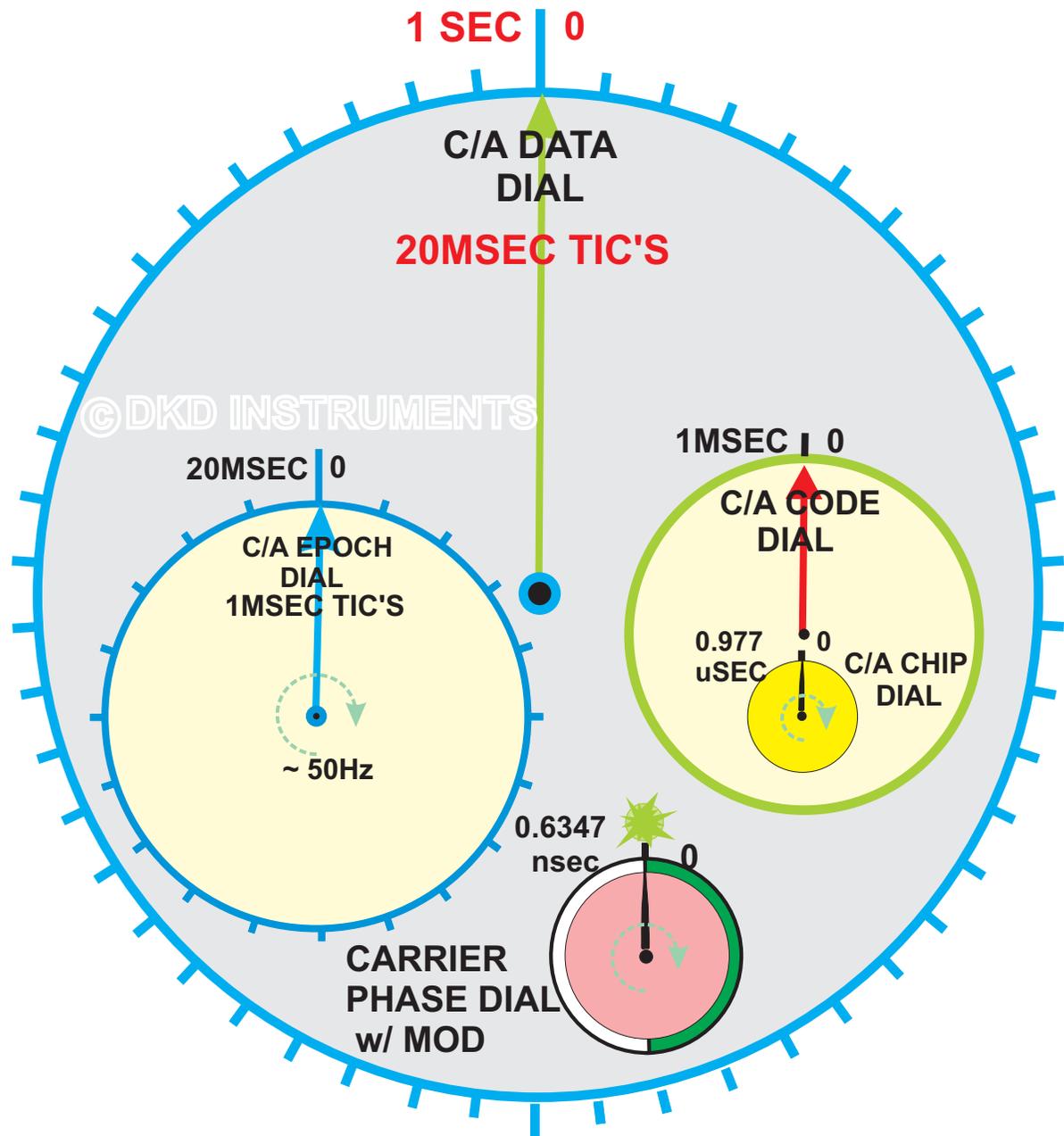


GPS L1 CLOCK TRANSMITTER ROTARY CLOCK MODEL AT THE SATELLITE, BEFORE TRANSMITTER

WITH THE DIFFERENT TYPES OF ENCODING WE HAVE DISCUSSED WE CAN NOW EXPLORE THE WAY IN WHICH OUR L1 MODEL SENDS ITS CLOCK INFORMATION TO A RECEIVER ON THE EARTH.

OUR GPS L1 MODEL TRANSMITTER ONLY HAS ONE LIGHT, ITS ON THE CARRIER PHASE DIAL. ALL THE OTHER DIALS PHASE INFORMATION IS ENCODED OR MODULATED ONTO THIS ONE DIAL/LIGHT.

ALL THE MODULATION IS BI-PHASE TYPE. NOT EVERY DIAL IS DIRECTLY ENCODED ONTO THE CARRIER PHASE DIAL. IN FACT OTHER THAN THE CARRIER PHASE DIAL ITSELF ONLY THE C/A CODE DIAL AND THE C/A DATA DIALS ARE DIRECTLY ENCODED ONTO THE CARRIER DIAL.



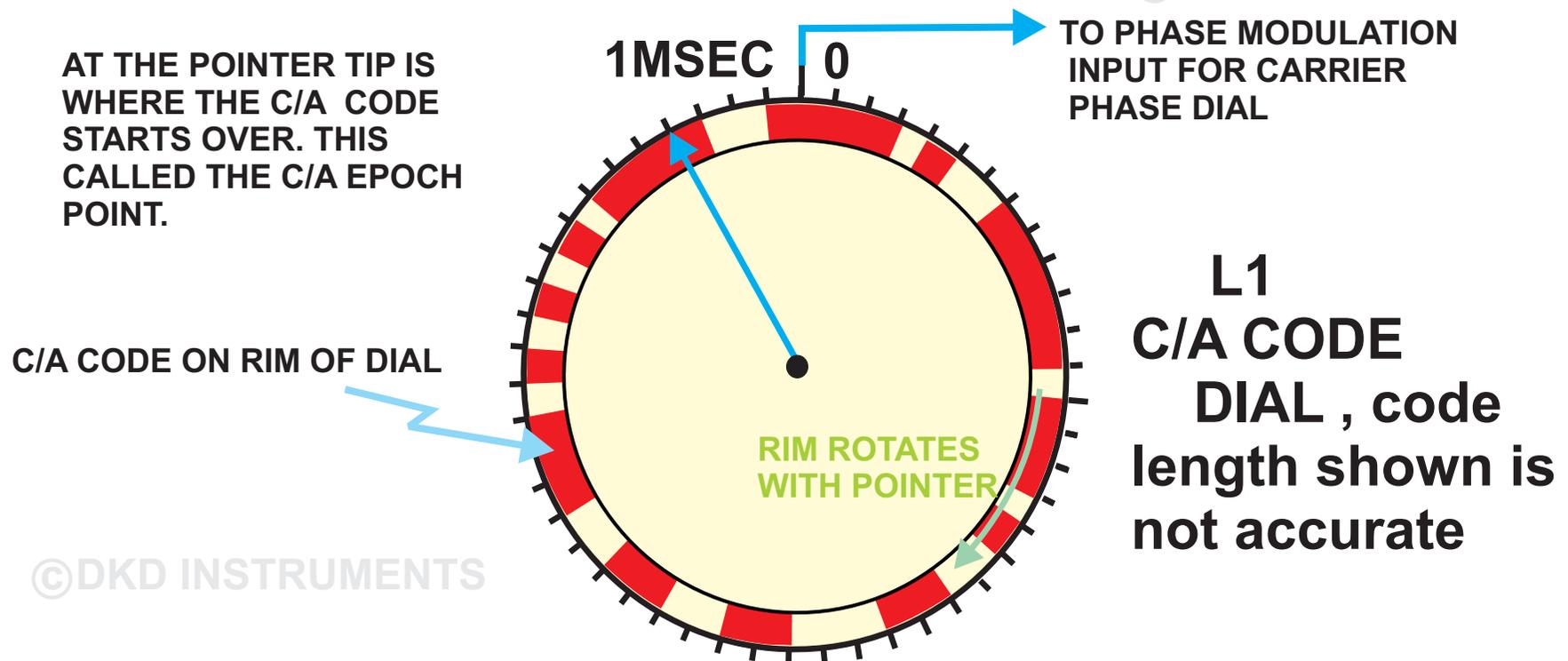
TRANSMITTING THE L1 GPS C/A CODE DIAL AND C/A CHIP DIALS

THE C/A CODE DIAL IS SETUP AS A PSEUDO RANDOM BINARY CODE OF LENGTH 1023 BITS. A NEW CODE BIT IS SENT AT EVERY TIC OF C/A CODE DIAL OR ONE CYCLE OF THE C/A CHIP DIAL.(NOT SHOWN). THE C/A CODE IS TOO LONG TO BE SHOWN CLEARLY SO THE IMAGE BELOW IS AN APPROXIMATION. THE CODE IS FIXED FOR EACH SATELLITE AND SERVES AS AN ADDRESS AS WELL AS A WAY TO SEND AN ENCODED CLOCK PHASE INFORMATION.

THE C/A CHIP DIAL DETERMINES THE RATE OF THE C/A CODE DIAL AND BY TRANSMITTING THE C/A CODE DIAL CODE WE ARE TRANSMITTING THE PHASE INFORMATION OF BOTH DIALS.

UNLIKE OUR PREVIOUS EXAMPLE OF SUCH A CODED DIAL THIS DIAL DOES NOT CONTROL A LIGHT DIRECTLY. INSTEAD IT SENDS ITS ENCODED OUTPUT TO THE PHASE MODULATOR OF THE CARRIER PHASE DIAL. AS BEFORE THE COLOR OF THE RIM AT THE ZERO TIMING MARK DETERMINES PHASE CONTROL OUTPUT.

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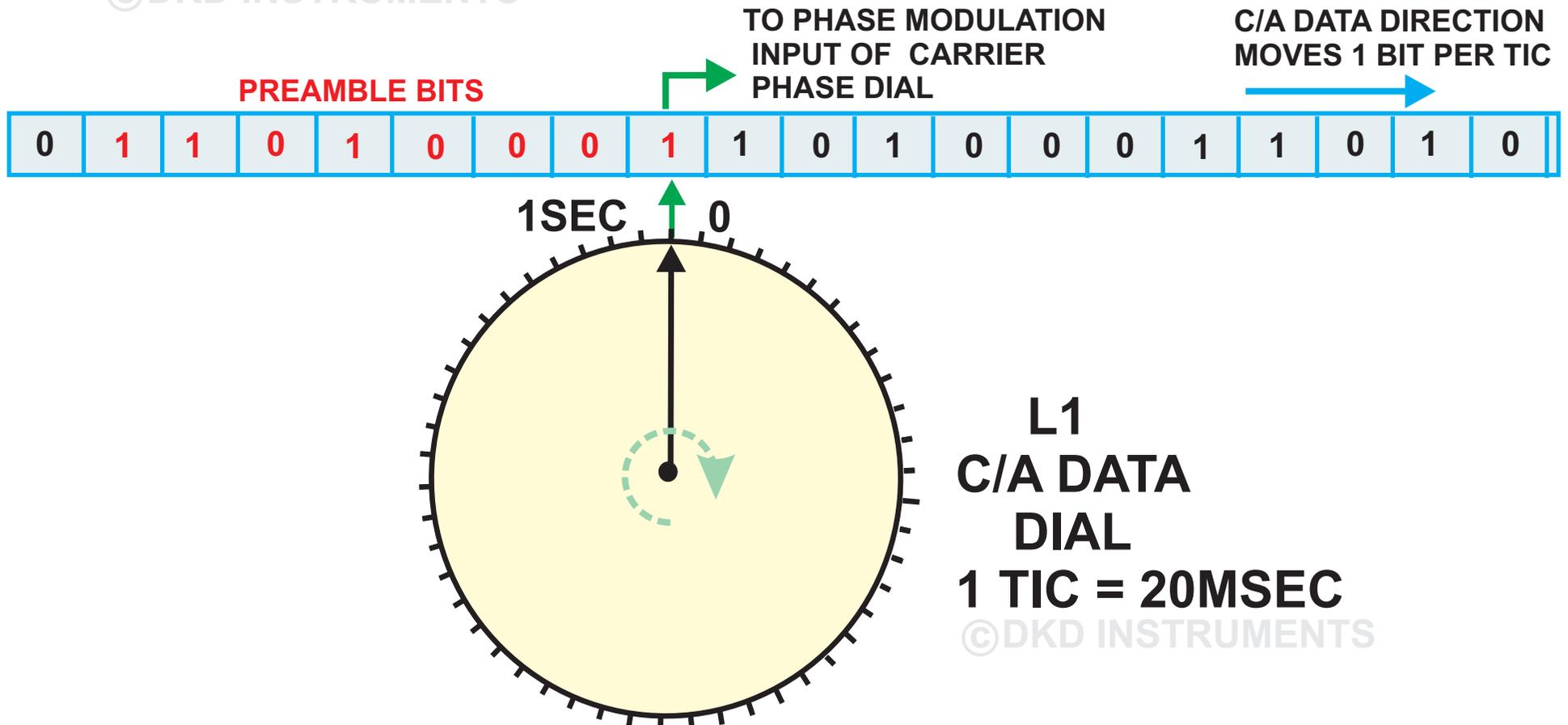


TRANSMITTING THE L1 GPS (50Hz) DATA DIAL

THE PHASE OF THE C/A DATA DIAL IS INDIRECTLY TRANSMITTED AS DATA BITS. THE L1 GPS SIGNAL SENDS A STREAM OF DATA TO RECEIVERS @ 50Hz RATE. THIS DATA IS ONLY ROUGHLY CYCLICAL SO IT IS INDICATED HERE AS LINEAR STREAM OF DATA THAT IS CLOCKED BY THE C/A DATA DIAL. AT EACH TIC THAT DATA BIT IS SENT TO THE PHASE MODULATION INPUT OF THE CARRIER PHASE DIAL.

EVERY 6 SECONDS THE DATA STREAM CONTAINS ANOTHER FRAME OF DATA THAT STARTS WITH A PREAMBLE SEQUENCE. WHEN THE RECEIVER FINDS THE PREAMBLE IT KNOWS THE FIRST BIT IS ON THE 1SECOND TIMING MARK OF THE C/A DATA DIAL.

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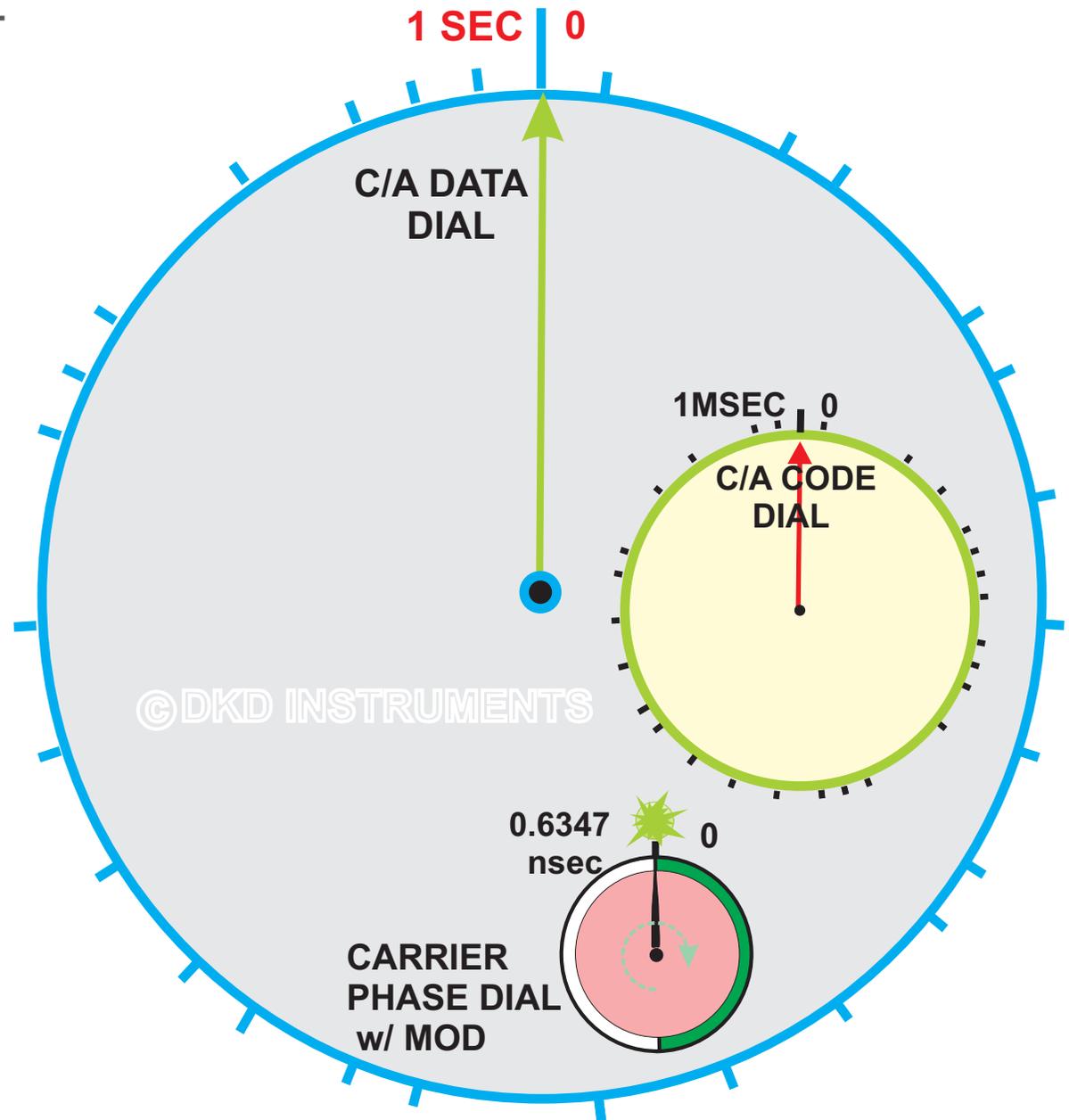


THE GPS L1 CLOCK INFORMATION THAT IS ACTUALLY TRANSMITTED

SHOWN HERE IS WHAT IS LEFT OF THE GPS L1 CLOCK AS SENT BY THE SATELLITE TO RECEIVERS ON EARTH.

THE 20MSEC DIAL IS COMPLETELY GONE. THE C/A CHIP DIAL IS GONE. TIMING MARKS ARE MISSING ON THE 1SEC AND C/A CODE DIALS. THIS OCCURS BECAUSE WE ARE NOT SENDING ALTERNATING 1'S AND 0'S, BUT INSTEAD THERE ARE PLACES WHERE THERE ARE STRINGS OF 1's OR 0's.

THE RECEIVER WILL HAVE TO RECONSTRUCT THE REPLICA CLOCK FROM THE INFORMATION THAT IS SENT. THE GOOD NEWS IS THE ZERO POINT TIMING MARKS OF THE REMAINING DIALS ARE SENT. WITH WHAT IS TRANSMITTED THE RECEIVER CAN REBUILD THE L1 CLOCK, BUT IT IS A CONVOLUTED AND COMPLEX PROCESS.

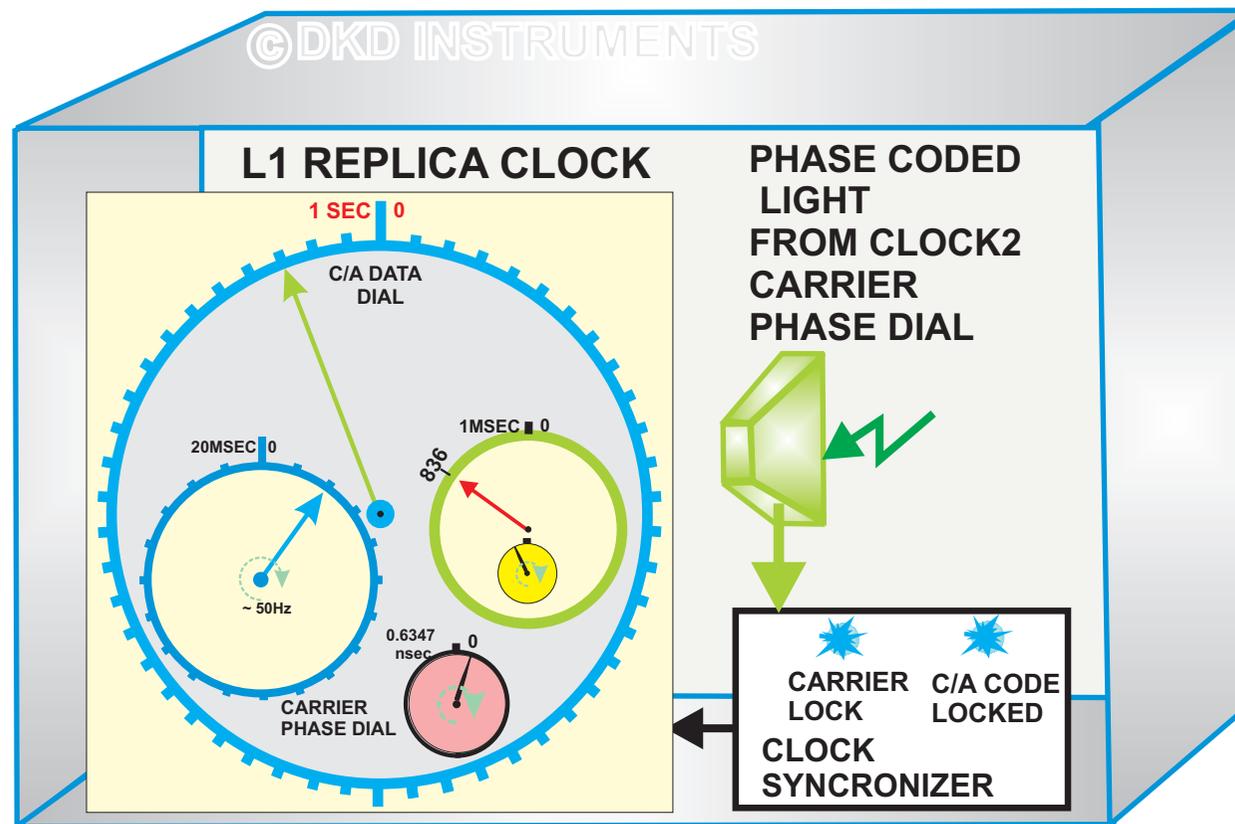


GPS L1 CLOCK RECEIVER & SYNCHRONIZER MODEL

WITH OUR CLOCK TRANSMITTER MODEL IN PLACE WE NEED TO ESTABLISH A MATING RECEIVER MODEL. SHOWN BELOW IS THE PORTION OF OUR MODELL1 RECEIVER THAT WILL TAKE TRANSMITTED CLOCK SIGNALS AND IMPRESS THEM ON THE REPLICA CLOCK. AS BEFORE THE SYNCHRONIZER SETS THE PHASE AND RATE OF ALL THE DIALS ON THE REPLICA CLOCK TO MATCH THE RECEIVED SIGNALS. OUR REPLICA CLOCK IS NOW A COPY OF THE COMPLETE 5 DIAL, L1 CLOCK.

L1 CLOCK SYNCHRONIZER/ RECEIVERS NEED TO HAVE THE ABILITY TO DEMODULATE THE CARRIER PHASE DIAL , PERFORM CORRELATION , RECOVER THE C/A CODE AND C/A CHIP DIALS AND RECOVER ALL THE MISSING DIAL INFORMATION FROM THE LIMITED TRANSMITTED CLOCK INFORMATION.

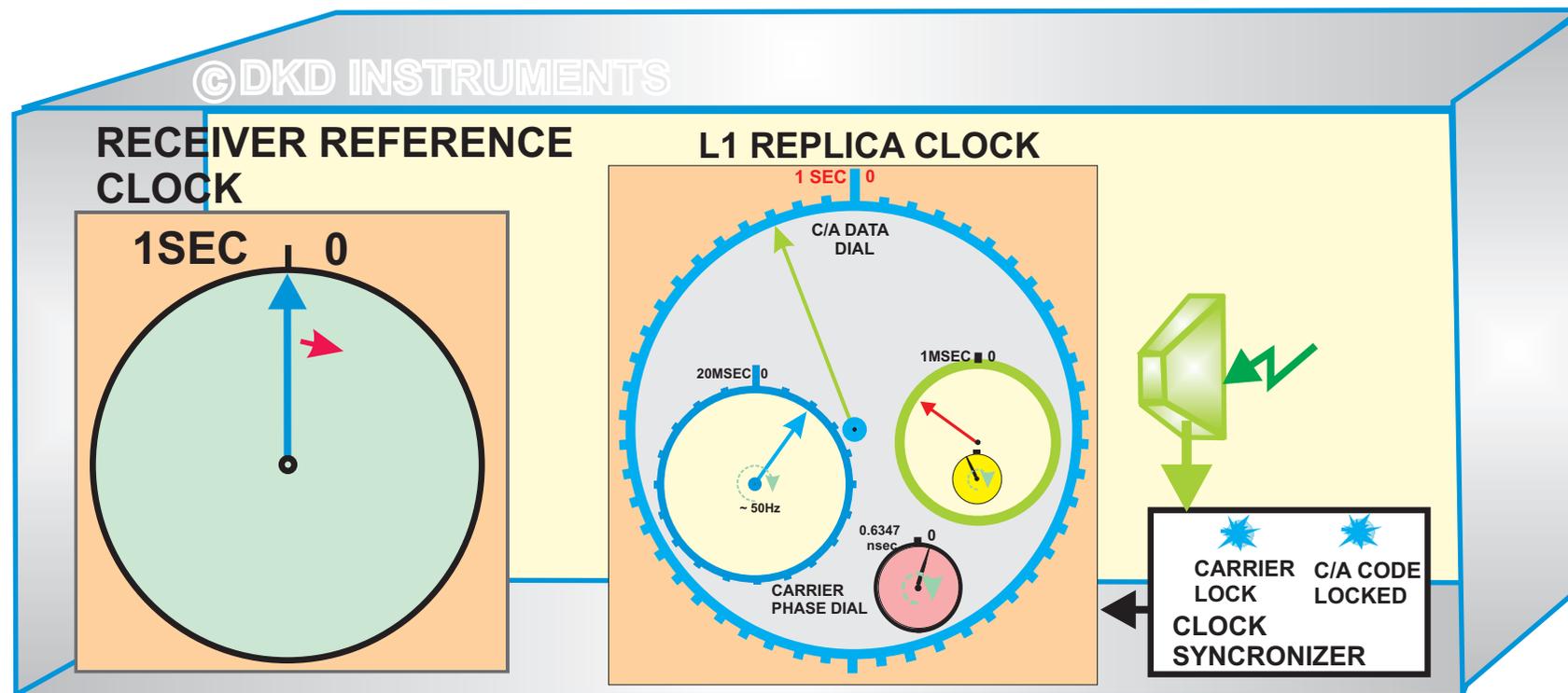
WHEN THE CODE AND CARRIER LOOPS ARE LOCKED AND THE ALL DIALS ON THE REPLICA CLOCK ARE RECOVERED THE RECEIVER IS READY TO MAKE RANGE MEASUREMENTS.



A COMPLETE GPS L1 RECEIVER MODEL USING LIGHT PULSES

TO COMPLETE OUR RECEIVER MODEL WE ADD A LOCAL RECEIVER REFERENCE CLOCK. THE REFERENCE CLOCK IS JUST OUR FAMILIAR CLOCK OF PREVIOUS MODELS. OUR COMBINED RECEIVER MODEL NOW HAS ALL THE ELEMENTS NEEDED TO RECEIVE DISTANT SATELLITE CLOCKS AND MAKE RANGE MEASUREMENTS BY COMPARISON TO THE REFERENCE CLOCK. NOW WE CAN SEE THE HEART OF GPS RECEIVERS: A REFERENCE CLOCK AND A REPLICA CLOCK FOR EACH SATELLITE TRACKED.

THE REFERENCE CLOCK SHOWN IS TYPICAL OF WHAT MANY L1 GPS RECEIVERS USE. IT IS A 0 TO 1 SEC DIAL WITH MILLIONS OF TIC'S, (TO MANY TIC'S TO SHOW). ANY PHASE AND RATE ERRORS OF THIS CLOCK WITH RESPECT TO TRANSMITTING SATELLITE CLOCK ARE USUALLY SOLVED FOR AS PART OF THE NAVIGATION SOLUTION.



A COMPLETE MODEL FOR RANGE MEASURING USING L1 SIGNAL STRUCTURE

WITH OUR RECEIVER AND TRANSMIT MODELS WE CAN NOW ASSEMBLE A COMPLETE SYSTEM FOR MEASURING RANGE FROM THE SATELLITE TO THE RECEIVER. THE TRANSMIT MODEL IS NOW PLACED ON THE SATELLITE. NOTE THAT OUR RECEIVERS REFERENCE CLOCK IS IN PHASE WITH THE SATELLITE CLOCK. THE PHASE STATE'S SHOWN ARE TAKEN EXACTLY WHEN THE RECEIVERS REFERENCE CLOCK HITS ITS ZERO TIMING MARK. WE CAN NOW SEE THE PHASE DIFFERENCE BETWEEN THE REPLICA CLOCK AND THE RECEIVERS REFERENCE CLOCK, WHICH IS A MEASURE OF RANGE FROM THE RECEIVER TO THE SATELLITE.

WE CAN MAKE THE *CODE PHASE* RANGE MEASUREMENT;

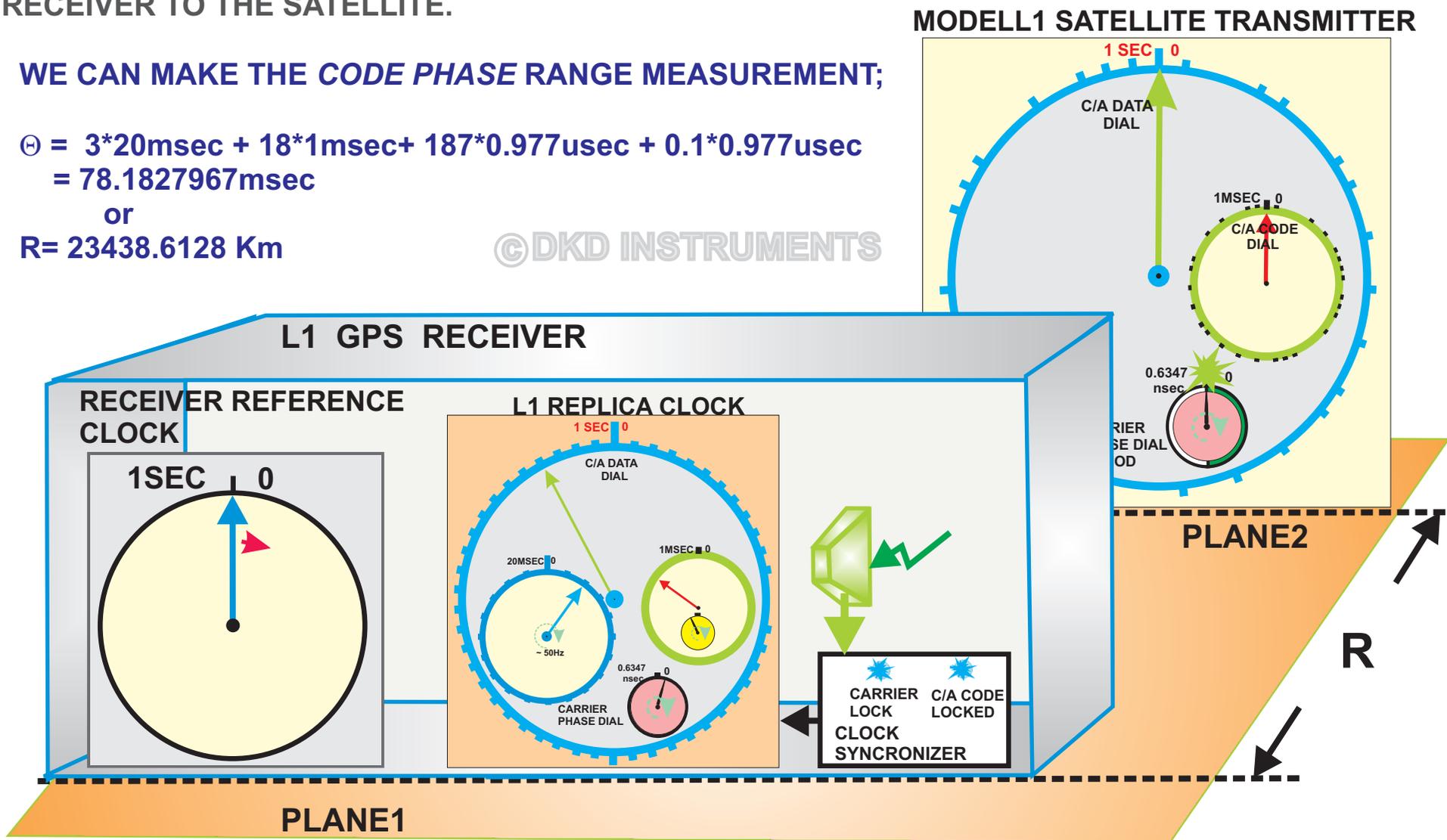
$$\Theta = 3 \times 20 \text{msec} + 18 \times 1 \text{msec} + 187 \times 0.977 \text{usec} + 0.1 \times 0.977 \text{usec}$$

$$= 78.1827967 \text{msec}$$

or

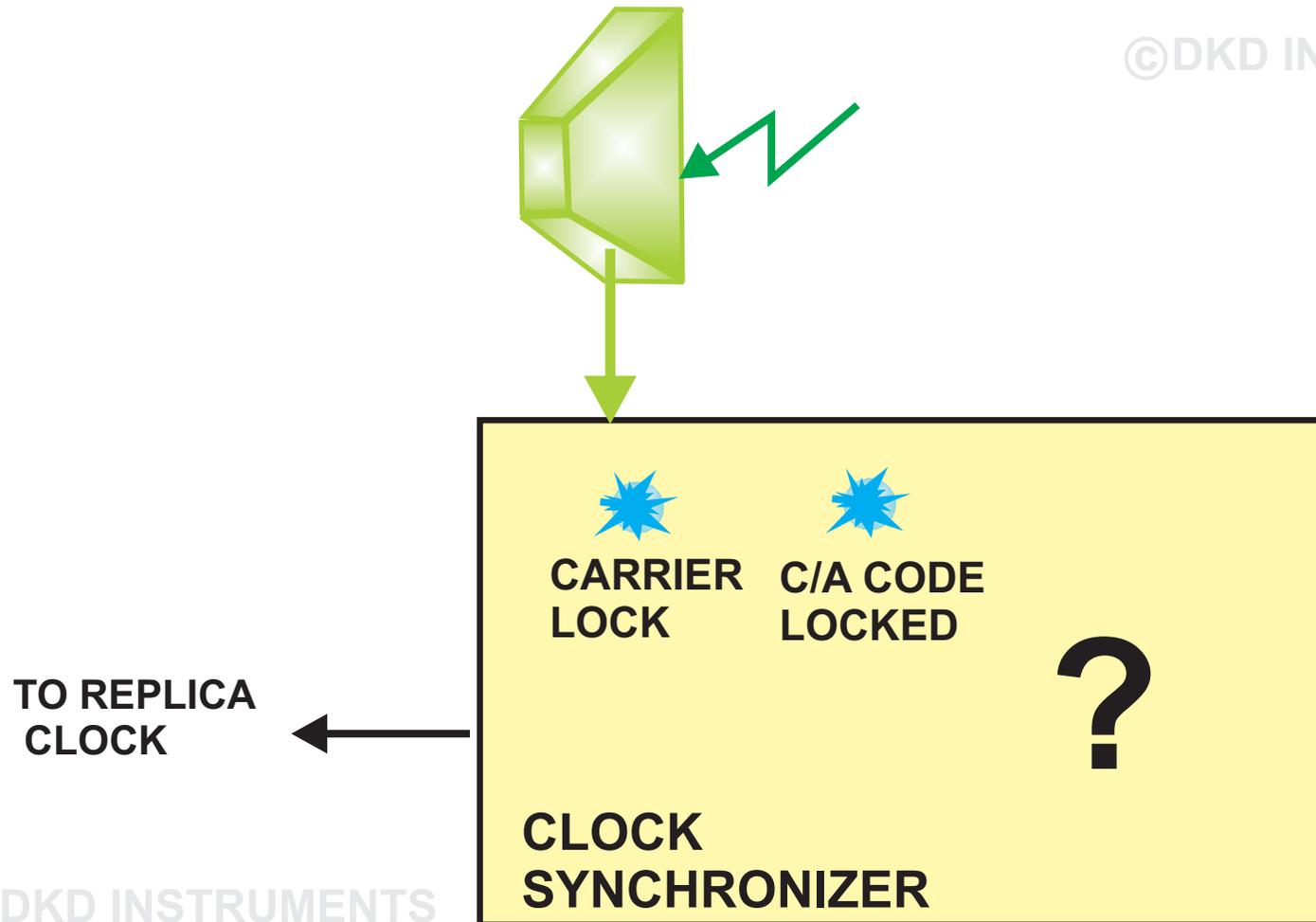
$$R = 23438.6128 \text{ Km}$$

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WE NOW SEE HOW OUR TRANSMITT AND RECEIVE L1 MODELS CAN BE USED TO MEASURE THE RANGE FROM THE SATELLITE TO THE RECEIVER. BUT THE MECHANISM, METHODS AND EVENTS OF JUST HOW THE RECEIVERS REPLICA CLOCK IS SET TO THE INCOMING TRANSMITTED CLOCK SIGNALS, i.e. THE CLOCK SYNCHRONIZER , IS STILL TO BE EXPLORED.

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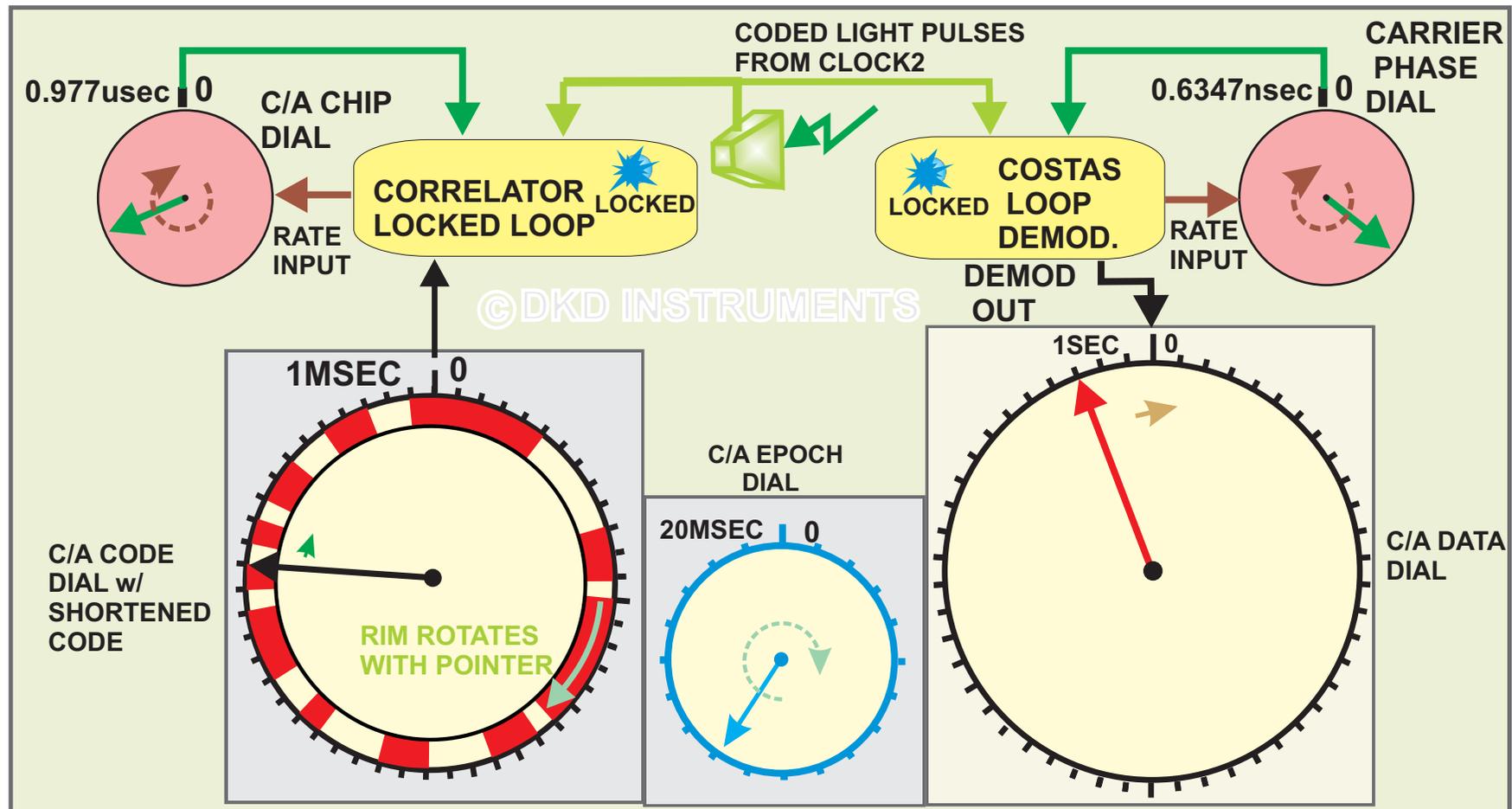


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L1 CLOCK SYNCHRONIZER, C/A CODE & CARRIER LOOPS

THE L1 C/A CLOCK RECEIVER/SYNCHRONIZER IS MADE OF TWO LOOPS, CODE AND CARRIER. WE CAN NOW SEE WHY THE C/A CHIP DIAL AND CARRIER PHASE DIALS ARE ANALOG IN OUR MODEL, THEY ARE THE RATE CONTROL POINTS FOR THE TWO LOOPS. THE CARRIER LOOP CANNOT ACHIEVE LOCK UNTIL CODE LOOP IS LOCKED. (A CORRELATION ISSUE).

UP TO THIS POINT WE HAVE SHOWN THE L1 RECEIVER REPLICIA CLOCK WITH ITS DIALS RECOVERED OR IN THE *TRACK MODE*. JUST AS BEFORE WE MUST PASS THRU THE ACQUISITION STAGE BEFORE THE TRACK STAGE CAN BEGIN. THE L1 ACQUISITION PROCESS IS COMPLEX AND HAPPENS IN A SEQUENCE OF STAGES OR STEPS.

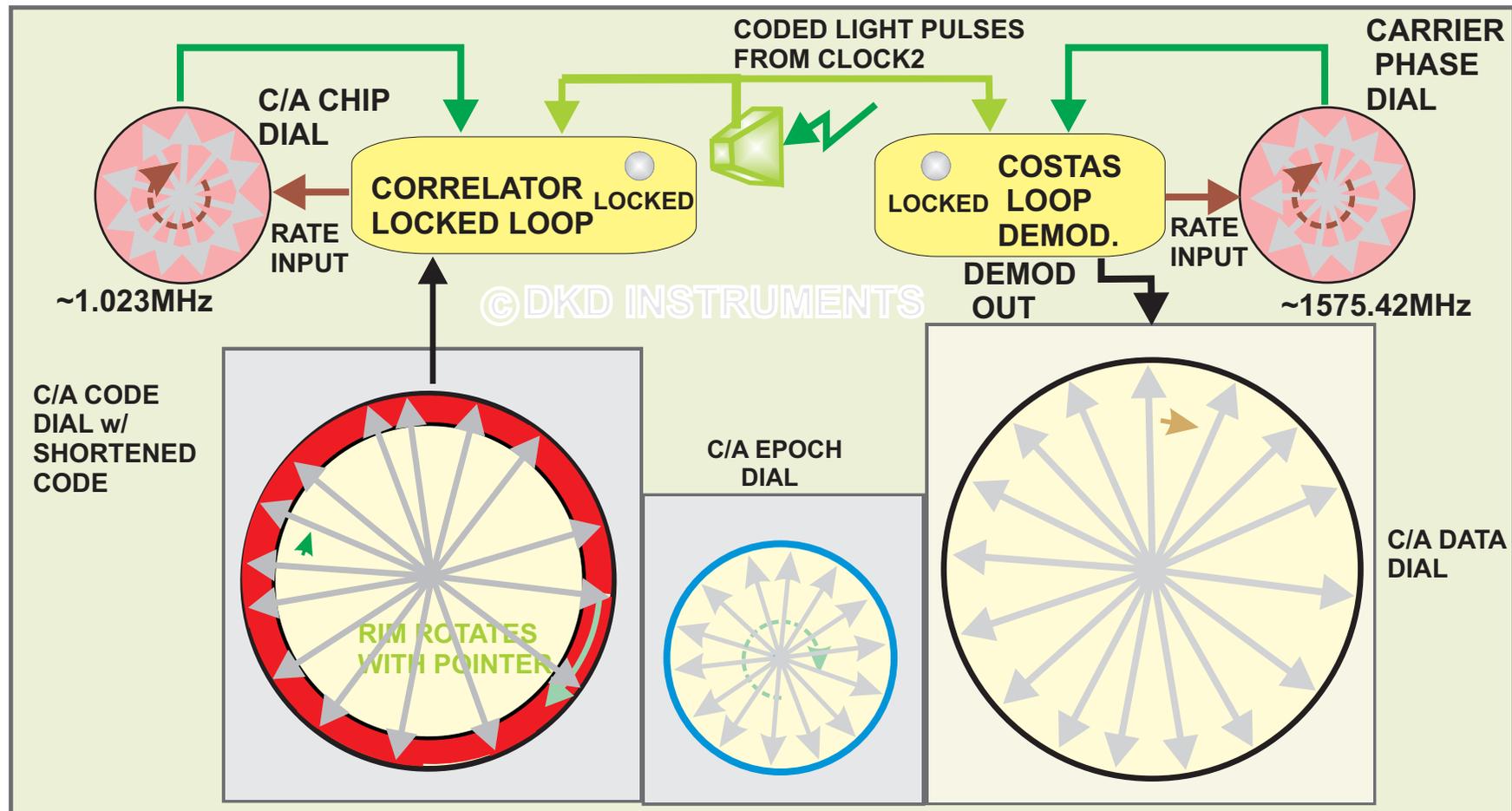


ALL PHASES TAKEN AT THE INSTANT REFERENCE CLOCK IS AT ZERO TIMING MARK

ACQUISITION STEP 1: ESTIMATE AND SET RATES OF CODE AND CARRIER DIAL'S

AT THIS STAGE NO PHASE INFORMATION IS KNOWN. TIMING MARKS ARE GONE, DIAL PHASES ARE UNKNOWN. THE C/A CODE AND CARRIER PHASE LOOPS ARE UNLOCKED. THE RECEIVER SET'S THE CODE RATE (C/A CHIP DIAL) TO NOMINAL. IT ALSO SWEEP'S OR SEARCHES THE POSSIBLE CARRIER PHASE DIAL RATES DUE TO UNKNOWN DOPPLER ON RECEIVED SIGNAL.

MODERN RECEIVERS HAVE MANY CORRELATOR'S TO SEARCH FOR CORRELATION FASTER. FFT'S MAY ALSO BE USED FOR BOTH CORRELATION AND CARRIER DIAL ACQUISITION .

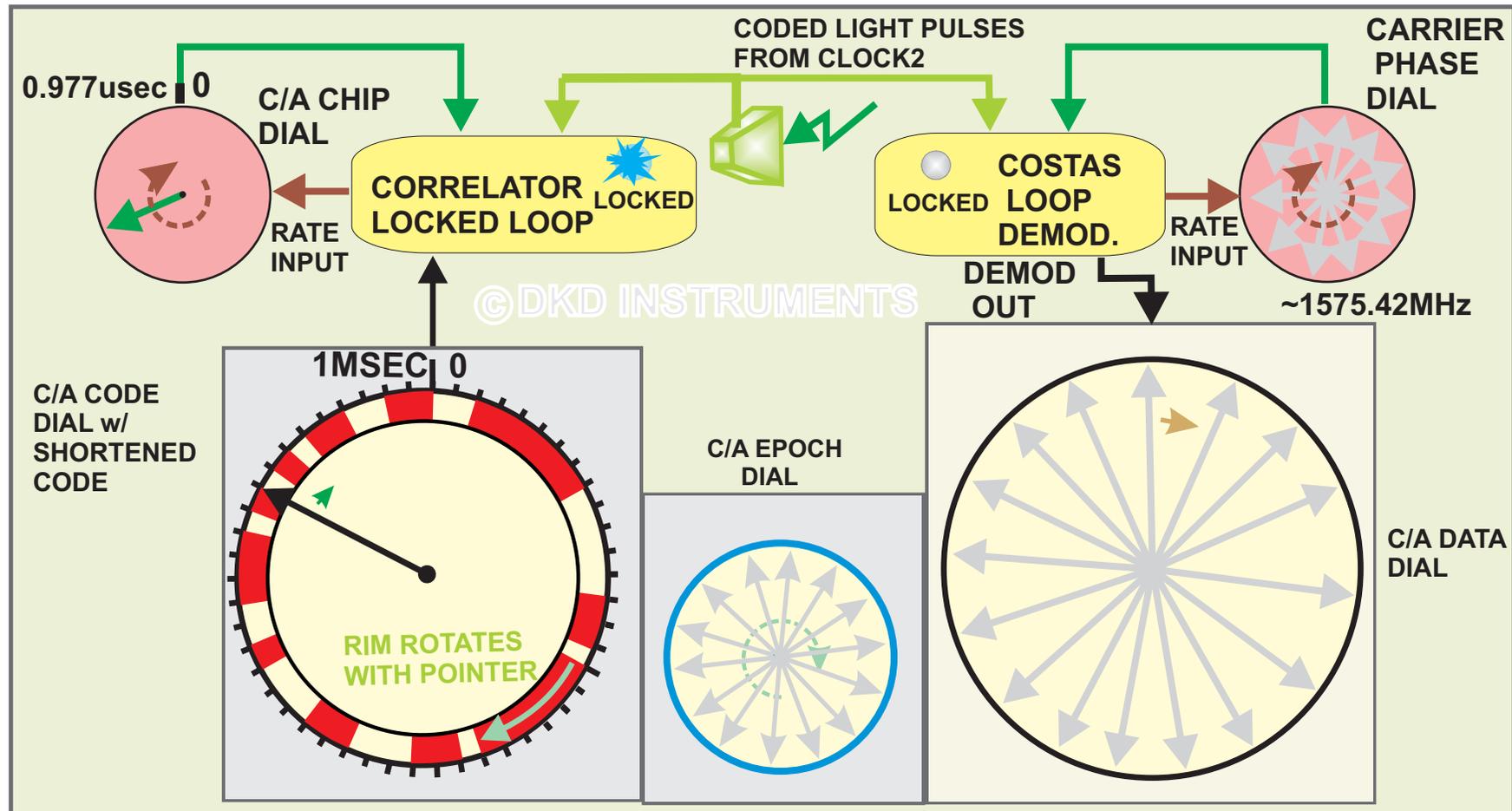


ALL PHASES TAKEN AT THE INSTANT REFERENCE CLOCK IS AT ZERO TIMING MARK

ACQ. STEP 2 : CORRELATION IS ACHIEVED C/A CHIP AND C/A CODE DIALS RECOVERED

CORRELATION IS FOUND AND THE CODE LOOP IS LOCKED. THE C/A CHIP AND THE C/A CODE DIAL'S PHASES NOW HAVE AN ESTIMATE OF THE PHASE DELAY TO THE SATELLITE, WITHIN THEIR RESOLUTION AND AMBIGUITY LIMITS.

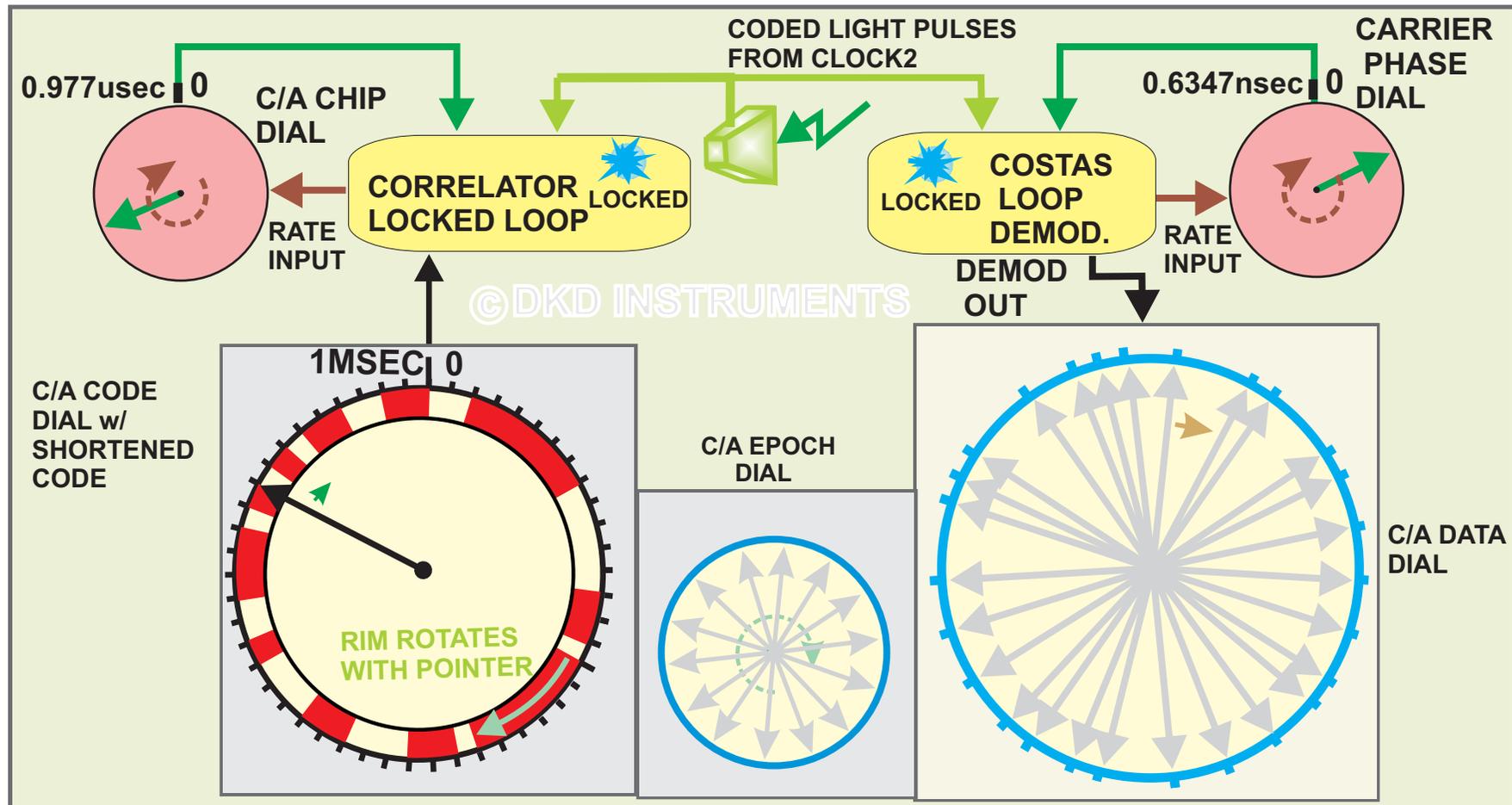
THE 20MSEC DIAL (C/A EPOCH DIAL) IS STILL UNKNOWN EVEN THOUGH WE HAVE THE ACCURATE 1KHZ C/A EPOCH PHASE. THIS IS SOMETIMES CALLED THE 1 OF 20 AMBIGUITY BECAUSE AT THIS STAGE IN THE ACQUISITION ITS IS NOT KNOWN WHICH OF 20 POSSIBLE C/A EPOCHS ALIGNS WITH ZERO TIMING MARK ON THE 20MSEC DIAL.



ALL PHASES TAKEN AT THE INSTANT REFERENCE CLOCK IS AT ZERO TIMING MARK

ACQ. STEP 3 : CARRIER PHASE LOOP IS LOCKED

THE CARRIER LOOP HAS CLOSED. THE PHASE OF THE CARRIER DIAL IS NOW RECOVERED. IN ADDITION THE DEMODULATED DATA FILLS IN SOME INFORMATION ON THE C/A DATA DIAL. THESE MARKS APPEAR WHERE EVER THE DATA CHANGES. WE STILL DON'T KNOW WHERE THE ZERO TIMING MARK IS ON THE 1SEC DIAL BUT WE ARE GETTING CLOSER. THE 20MSEC DIAL PHASE IS STILL IN QUESTION.

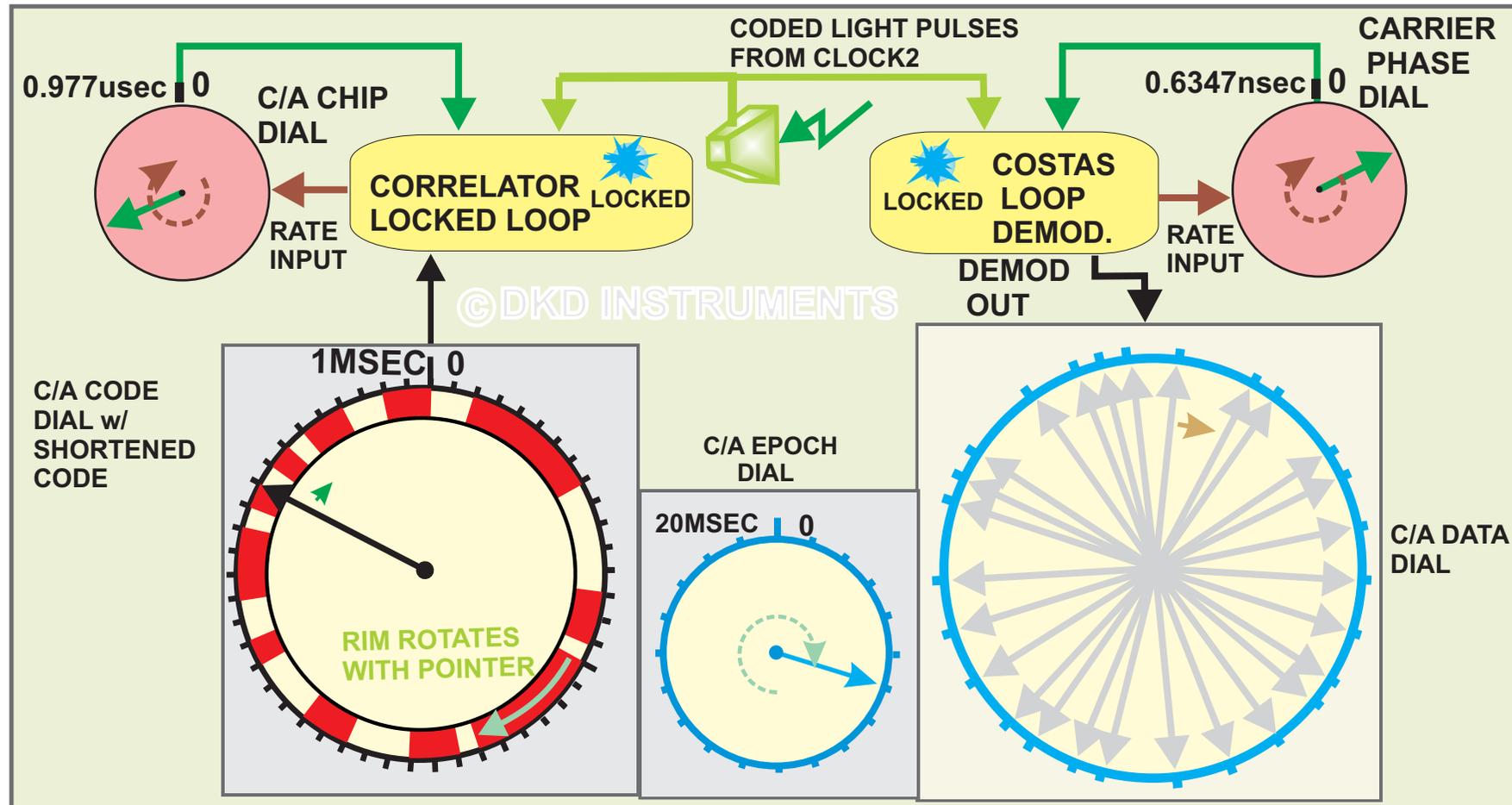


ALL PHASES TAKEN AT THE INSTANT REFERENCE CLOCK IS AT ZERO TIMING MARK

ACQ. STEP 4 : 20MSEC DIAL aka C/A EPOCH IS RECOVERED

WITH THE HELP OF THE 1SEC DIAL WE CAN RECOVER THE PHASE OF THE 20MSEC DIAL. THE DATA DEMODULATION OF THE CARRIER PHASE LOOP PROVIDED PHASE INFORMATION ON THE 1SEC DIAL. IN PARTICULAR WHERE EVER THE DATA CHANGED FROM A ZERO TO A ONE OR FROM A ONE TO A ZERO A TIC MARK MUST BE THERE ON THE 1SEC DIAL.

WHERE EVER THERE IS A TIC MARK ON THE 1SEC DIAL (C/A DATA DIAL) THE 20MSEC DIAL MUST BE AT ITS ZERO TIMING MARK. THUS THE 50HZ DATA EDGES PROVIDED BY THE COSTAS LOOP DEMODULATOR ALLOW THE RECOVERY OF THE PHASE OF THE 20MSEC DIAL.



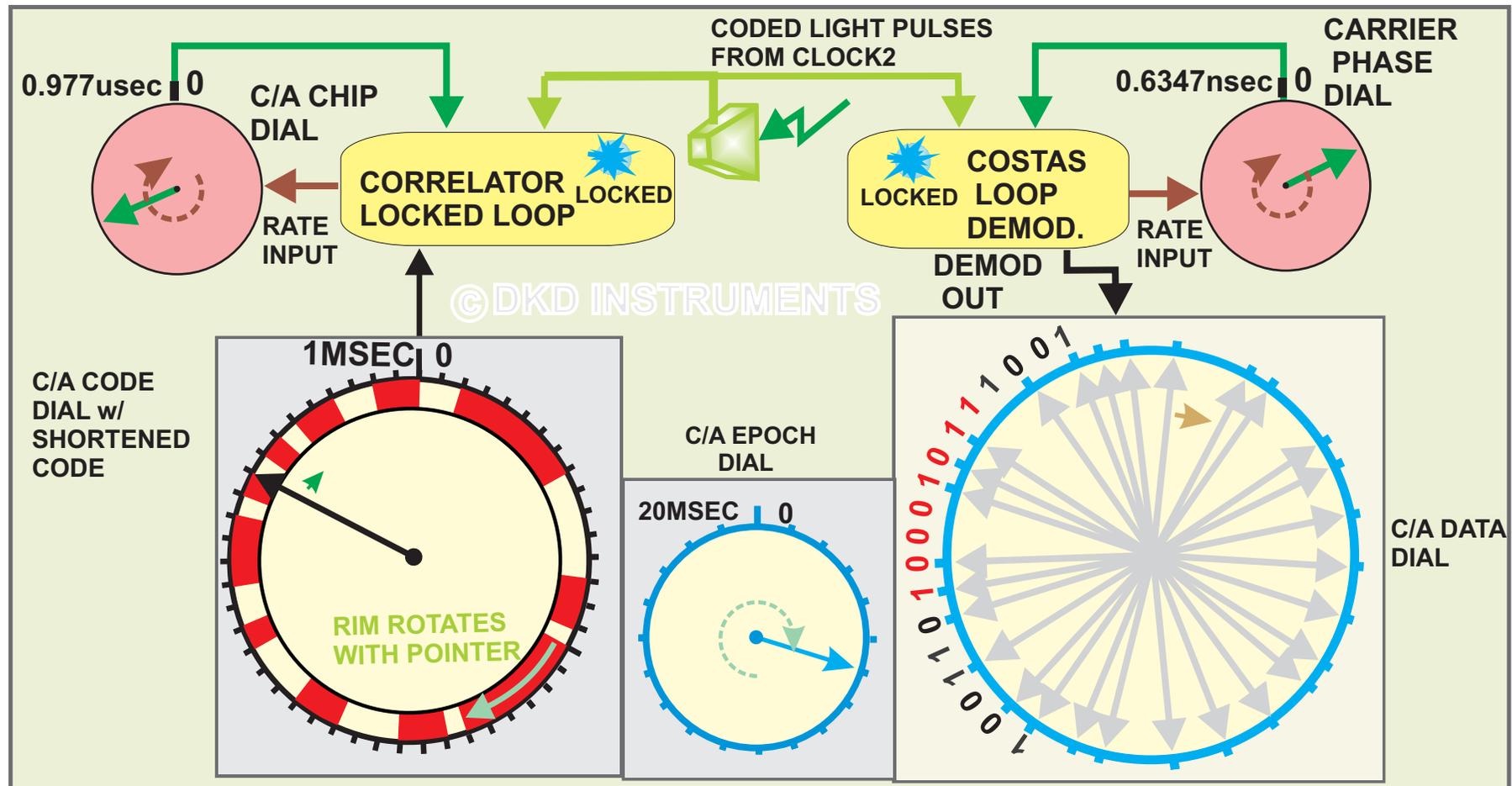
ALL PHASES TAKEN AT THE INSTANT REFERENCE CLOCK IS AT ZERO TIMING MARK

ACQ. STEP 5 : FINDING THE 1SEC DIAL ZERO TIMING MARK

TO RECOVER THE 1 SEC DIAL PHASE WE NEED TO KNOW WHICH ONE OF 50 POSSIBLE CYCLES OF THE 20MSEC DIAL CORRESPONDS TO THE 1SEC ZERO TIMING MARK. THIS IS SOMETIMES CALLED THE 1 OF 50 AMBIGUITY.

WE HAVE THE DEMODULATED DATA AND CAN SEE SOME TIMING MARKS, WHERE THE DATA EDGES ARE. WE NEED TO FIND THE 8 BIT PREAMBLE IN THE 50Hz DATA. THIS WILL INDICATE WHICH CYCLE OF 20MSEC DIAL THE 1 SEC ZERO TIMING MARK OCCURS.

SOME OF THE 50Hz DATA IS SHOWN ON THE 1SEC DIAL. THE PREAMBLE IS INDICATED IN RED. THE FIRST BIT OF THE PREAMBLE OCCURS ON THE 1SEC TIMING MARK.



ALL PHASES TAKEN AT THE INSTANT REFERENCE CLOCK IS AT ZERO TIMING MARK

ACQ. STEP 5, CONTINUED : RECOVERING THE 1SEC DIAL PHASE, DONE

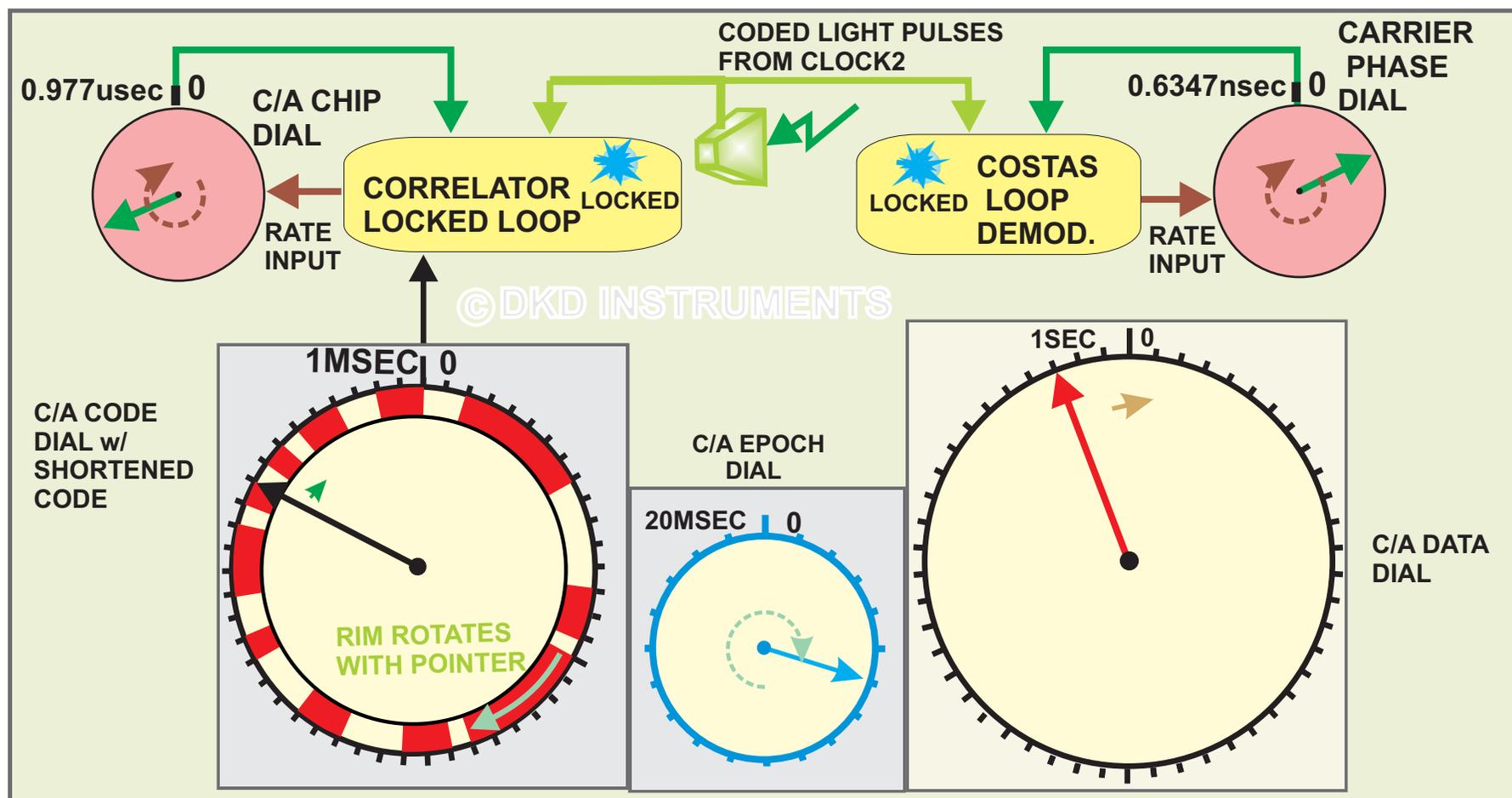
WITH THE PREAMBLE FOUND WE CAN RECOVER THE 1SEC DIAL PHASE AND ALL ITS TIMING MARKS. NOTE THAT AFTER THE 1 SEC DIAL PHASE IS SET THE DEMODULATED DATA INPUT TO THIS DIAL IS NO LONGER NEEDED. THE PHASE WILL REMAIN CORRECT AS LONG AS THE DIALS PRECEDING THIS DIAL REMAIN CORRECT IN RATE AND PHASE.

WITH ALL DIALS RECOVERED WE CAN NOW MAKE THE CODE PHASE RANGE MEASUREMENT OF ;

$$\Theta = 3*20\text{msec} + 14*1\text{msec} + [9/50]*[1023]*0.977\text{usec} + 0.66*0.977\text{usec} \Rightarrow 74.184784\text{msec}$$

or

$$\text{RANGE} = 22240.038\text{Km}$$



ALL PHASES TAKEN AT THE INSTANT REFERENCE CLOCK IS AT ZERO TIMING MARK

SUMMARY OF PART II

- ▶ A LIGHT PULSE TRANSMITTER MODEL CAN BE USED IN LIEU OF RADIO WAVE TRANSMITTER
- ▶ THE BASIC LIGHT PULSE RECEIVER IS COMPOSED OF A REPLICA CLOCK, CLOCK SYNCHRONIZER AND LIGHT SENSORS.
- ▶ THE REPLICA CLOCK IS TYPICALLY A COPY OF THE COMPLETE TRANSMIT CLOCK
- ▶ THE SYNCHRONIZER TAKES THE RECEIVED CLOCK SIGNALS AND ADJUSTS REPLICA CLOCK RATE AND PHASE TO MATCH A DELAYED IMAGE OF THE TRANSMIT CLOCK
- ▶ CLOCK SYNCHRONIZERS ARE COMPOSED OF PHASE COMPARISON COMBINED WITH A FEEDBACK LOOP SO AS TO CONTINUALLY ADJUST REPLICA CLOCK RATE OR PHASE SO AS TO NULL PHASE ERROR wrt RECEIVED CLOCK SIGNAL PHASE.
- ▶ THE PROCESS OF GETTING A REPLICA CLOCK ALIGNED IN PHASE AND RATE TO TRANSMITTED CLOCK SIGNALS IS CALLED ACQUISITION
- ▶ THE PROCESS OF KEEPING THE REPLICA CLOCK ALIGNED TO INCOMING CLOCK SIGNALS FROM TRANSMITTER IS CALLED TRACK OR TRACKING
- ▶ THE GPS L1 SIGNAL IS COMPOSED OF A REDUCED AND ENCODED SET OF CLOCK SIGNALS
- ▶ NOT ALL DIALS ARE SENT AND TIMING MARK INFORMATION IS ALSO OMITTED FROM SOME DIALS THAT ARE SENT
- ▶ ALL THE CLOCK SIGNALS ARE ENCODED ONTO ONE SOURCE, A PHASE MODULATED CARRIER.
- ▶ THE C/A CODED DIAL CAN ONLY BE RECOVERED BY CORRELATION METHODS
- ▶ THE C/A CODED DIAL MUST BE RECOVERED BEFORE THE CARRIER DIAL (SEE FAQ)
- ▶ C/A CODE MUST BE KEPT WITHIN ONE CHIP OF PROPER ALIGNMENT OTHERWISE CORRELATION LOOP IS UNLOCKED
- ▶ THE C/A CODE ALSO ACTS AS AN ADDRESS, EACH SATELLITE HAS ITS OWN ASSIGNED C/A CODE.
- ▶ WHEN WE COMBINE A REFERENCE CLOCK WITH THE REPLICA CLOCK AND A CLOCK SYNCHRONIZER WE HAVE THE BASIC MODEL FOR A SINGLE CHANNEL GPS RECEIVER
- ▶ FOR EVERY ADDED CHANNEL A REPLICA CLOCK AND ITS SYNCHRONIZER MUST BE ADDED
- ▶ ONE CHANNEL CAN TRACK ONE SATELLITE SIGNAL
- ▶ THE REFERENCE CLOCK SERVES AS THE PHASE AND RATE REFERENCE FOR ALL CHANNELS
- ▶ ALL PHASE (OR RANGE , OR DELAY) AND RANGE RATE (DOPPLER) ARE DERIVED FROM PHASE COMPARISONS OF THE REPLICA CLOCK TO THE REFERENCE CLOCK
- ▶ ANY ERRORS OF REFERENCE CLOCK wrt SATELLITE CLOCK CAN APPEAR IN RANGE OR RANGE RATE MEASUREMENTS (IF NOT ACCOUNTED FOR).

FAQ for PART II

Q] WHEN A LOWER DIAL IS LOCKED , SUCH AS THE C/A CHIP DIAL, SHOULDN'T THE SUBSEQUENT DIALS PHASE BE STABLE wrt REFERENCE CLOCK PHASE? THE GRAPHICS SHOW A PHASE THAT IS ALL OVER THE PLACE.

A] THE IMAGES SHOWING INDETERMINATE PHASE OF DIALS WHEN A LOWER DIAL IS LOCKED ARE DRAWN THAT WAY TO CONVEY WHAT THE PHASE WOULD BE FOR MANY TRIALS OF THE ACQUISITION PROCESS, NOT JUST THE ONE THAT WAS BEING DISCUSSED. IN SUCH A SERIES OF TRIALS MANY POTENTIAL PHASES COULD RESULT THAT ARE STABLE wrt TO REFERENCE BUT ARE INCORRECT AS THEY HAVE NOT BEEN SYNC'ED TO INCOMING CLOCK INFORMATION.

Q] SHOULDN'T THE RATE OF THE DIALS CHANGE WITH DOPPLER AND HENCE THE PERIOD USED IN THE RANGE COMPUTATIONS?

A] NO. EVEN THOUGH FOR THE OBSERVER OF A DISTANT CLOCK, A CLOCK THAT IS MOVING wrt TO OBSERVER, THE OBSERVER CONCLUDES ITS RATE IS SLOW OR FAST. BUT THE RATE AT THE MOVING CLOCK HAS NOT CHANGED FOR AN OBSERVER MOVING WITH THAT CLOCK. THE APPARENT RATE DIFFERENCE IS REALLY DUE TO A CONTINUOUS PHASE CHANGE. WHAT WE ARE MEASURING IS THE PHASE DIFFERENCE OF THE REPLICA CLOCK (OR DELAYED IMAGE) AGAINST THE DISTANT CLOCK ,THEREFORE THE PHASE SCALE OF THE REPLICA CLOCK AND THE DISTANT CLOCK MUST BE THE SAME.

Q] WHY DOES CORRELATION LOCK HAVE TO OCCUR BEFORE CARRIER PHASE LOCK IN THE L1 RECEIVER?

A] THE ANSWER TO THIS QUESTION GOES BEYOND WHAT CAN EASILY, IF AT ALL , SHOWN WITH OUR ROTARY CLOCK MODEL. IF WE USE THE DELAYED IMAGE MODEL ONE COULD SAY THAT THE INCOMING CLOCK IS NOT EVEN VISIBLE UNTIL CORRELATION IS ACHIEVED. CORRELATION WILL RETRIEVE THE HIDDEN CLOCK AND MAKE IT "SEEABLE". THE C/A CODE LOCK PROCESS IS IN A SENSE A BLIND SEARCH. ONCE CORRELATION IS FOUND INFORMATION ON THE REMAINING CLOCK DIALS CAN BE RECEIVED OR "SEEN". ONCE WE CAN SEE IT WE CAN TRY AND LOCK THE CARRIER LOOP.

FAQ for PART II, continued

Q] ANALOG CLOCKS ARE TYPICALLY NOT SEEN IN MODERN RECEIVERS. WHY DO YOU USE AN ANALOG CLOCK DIAL MODEL FOR C/A CHIP AND CARRIER PHASE DIALS?

A] TRUE , MOST RECEIVERS ARE ALL DIGITAL. MOST USE A NCO TYPE CLOCK FOR THE CARRIER PHASE AND C/A CHIP DIALS. THAT TYPE OF CLOCK USUALLY HAS AT LEAST 24BITS OF PHASE RESOLUTION. AT THAT LEVEL OF RESOLUTION IT IS VIRTUALLY AN ANALOG DIAL.

Q] WHY IS THE RESOLUTION OF THE C/A CHIP AND CARRIER PHASE DIALS TYPICALLY LIMITED TO $\sim 1/100$ OF A CYCLE ?

A] THE TWO ANALOG DIALS OF THE REPLICATED CLOCK , C/A CHIP AND CARRIER PHASE, ARE UNDER PHASE LOCK TYPE CONTROL LOOPS. BOTH OF THESE LOOPS HAVE RECEIVER NOISE IN THEM. THIS NOISE GETS ONTO THE PHASE OF THE DIALS BEING LOCKED. THE NET RESULT IS EVEN THOUGH, IN PRINCIPLE, THE RESOLUTION IN PHASE IS VERY HIGH THE ACHIEVABLE PRECISION IS MUCH LOWER DUE TO THESE PHASE DISTURBANCES.

Q] WHY IS THE C/A CHIP DIAL PHASE NOT TIED DIRECTLY TO THE CARRIER PHASE DIAL?

A] THE NOISE PRESENT ON THE C/A CHIP DIAL LIMITS ITS PHASE ACCURACY TO $\sim 1/100$ CYCLE. THAT IS ABOUT 3 METERS OF RANGE. THE NUMBER OF CARRIER PHASE CYCLES IN THAT RANGE ARE ABOUT 15, USING 1 CARRIER CYCLE ~ 0.2 METERS ($3\text{M}/0.2\text{M} = 15$). THIS MEANS WE DO NOT KNOW WHICH ONE OF APPROXIMATELY 15 CYCLES OF THE CARRIER PHASE DIAL OCCURS AT THE C/A CHIP DIAL ZERO TIMING MARK. IN ADDITION THE CARRIER PHASE TRACKING LOOP CAN HAVE CYCLE SLIPS. IF WE DID TIE THE C/A CHIP DIAL TO THE CARRIER PHASE DIAL BY DIRECT GEAR RATIO WHAT WOULD HAPPEN IS WE WOULD LIKELY HAVE SOME INITIAL PHASE ERROR IN C/A CODE DIAL PHASE AND IT WOULD LIKELY GROW WORSE DUE TO CYCLE SLIPS. EVENTUALLY A SLIP OF 1 C/A CHIP COULD OCCUR AND AT THAT POINT THE CORRELATION LOOP WOULD UNLOCK AND OUR REPLICATED CLOCK IS NOW TOAST. THIS PARTICULAR ISSUE IS WHY THERE ARE *CARRIER AIDED C/A CODE TRACKING* LOOPS. SUCH LOOPS USE THE CARRIER PHASE DIAL IN A WAY THAT OPTIMIZES C/A TRACKING WITHOUT GOING SO FAR AS PERMANENT DIRECT DRIVE OF C/A CHIP DIAL FROM CARRIER PHASE DIAL.

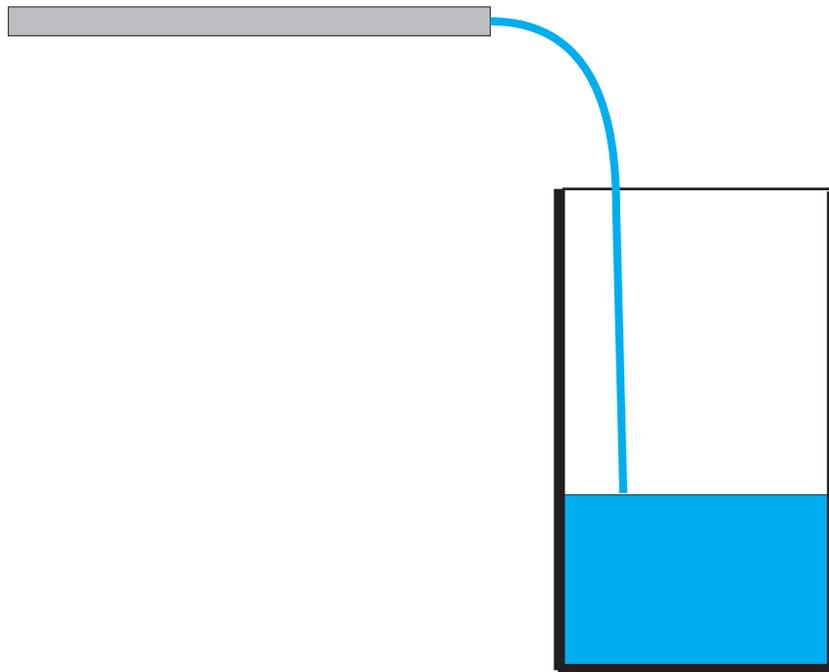
APPENDIX I

FUNDAMENTALS OF CLOCKS



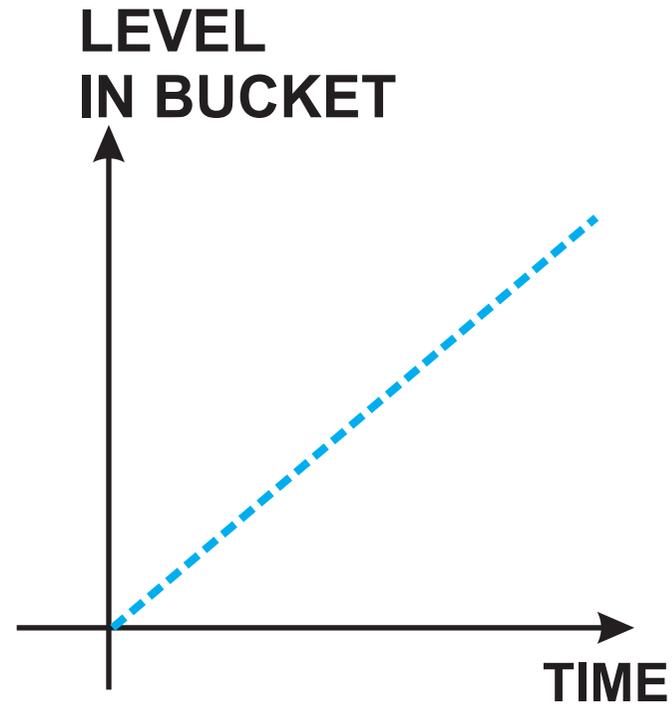
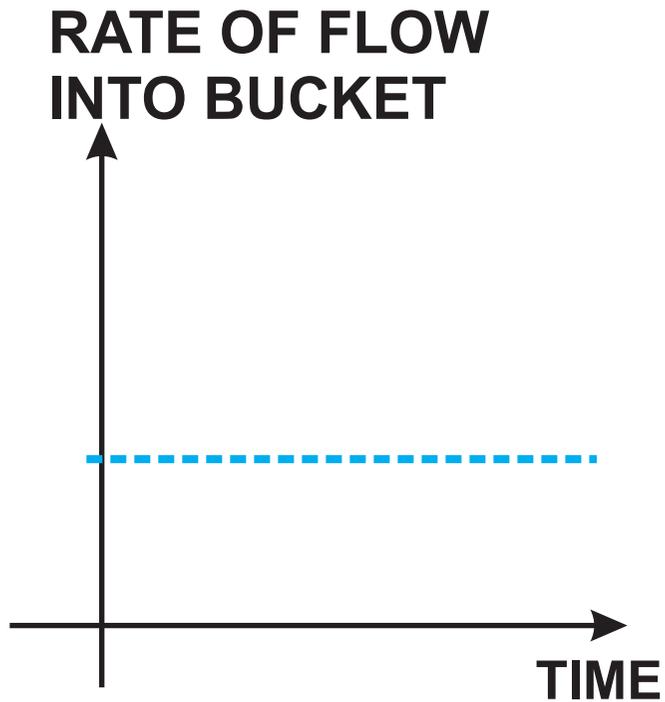
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A SIMPLE WATER CLOCK



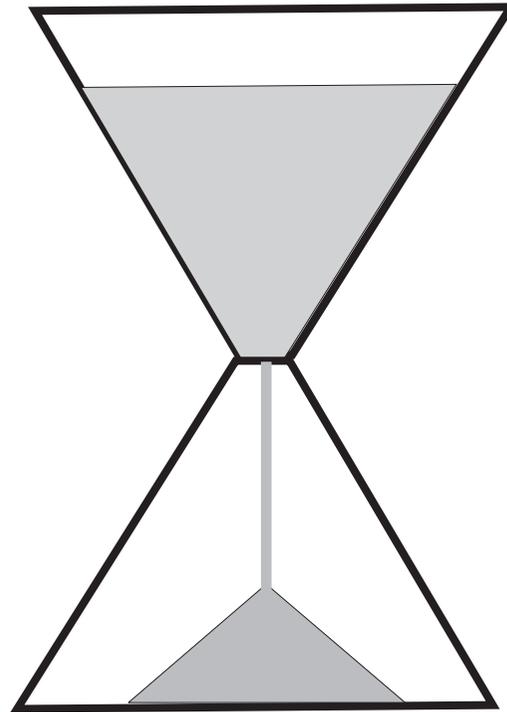
**WATER FLOWS
INTO THE BUCKET AT
A CONSTANT RATE.
LEVEL IN BUCKET IS
MEASURE OF TIME**

RELATION BETWEEN LEVEL IN BUCKET AND RATE OF FLOW INTO BUCKET

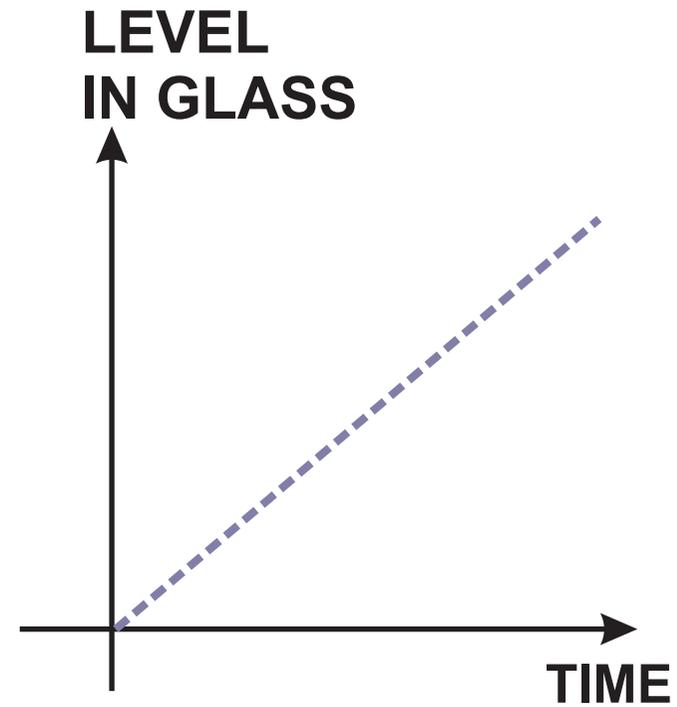
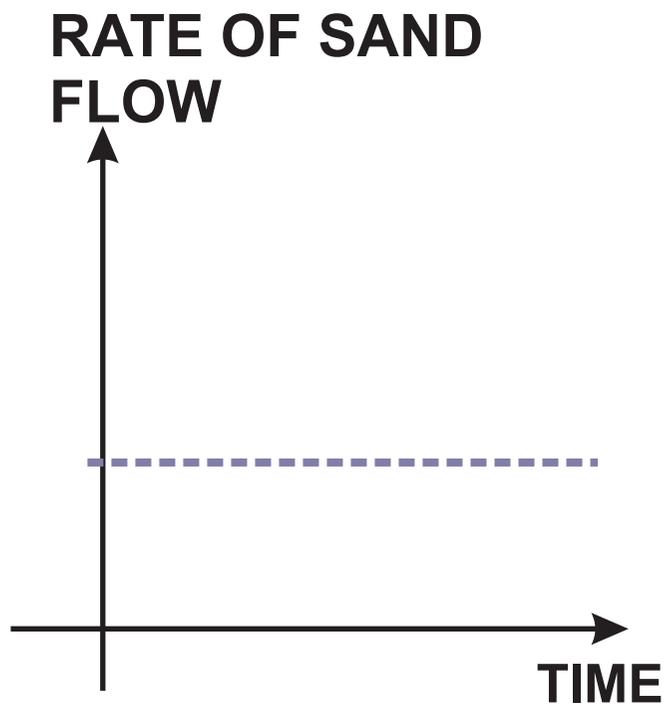


HOUR GLASS CLOCK

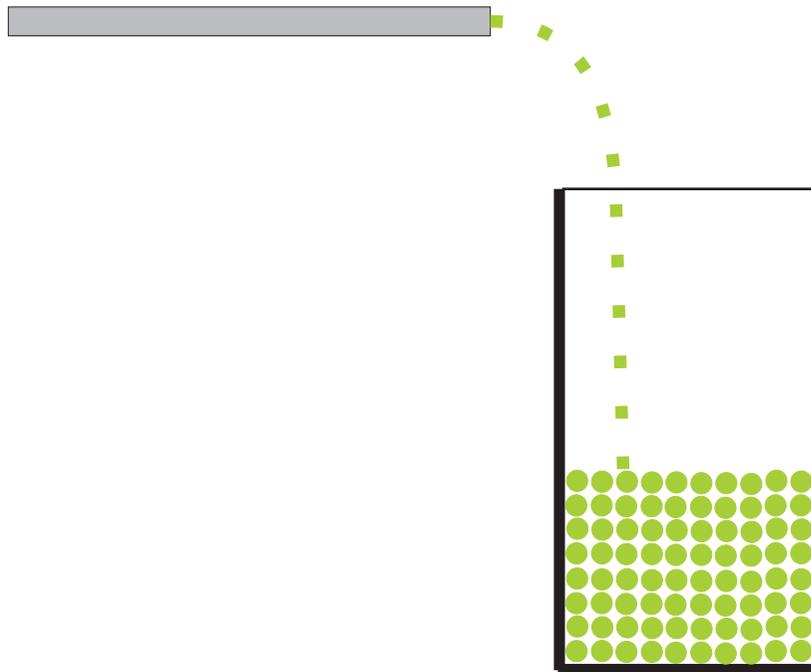
**A CLASSIC HOUR GLASS CLOCK WITH
FLOWING SAND, SAND LEVEL MEASURES TIME**



RELATION BETWEEN LEVEL IN BOTTOM OF GLASS AND RATE OF SAND FLOW

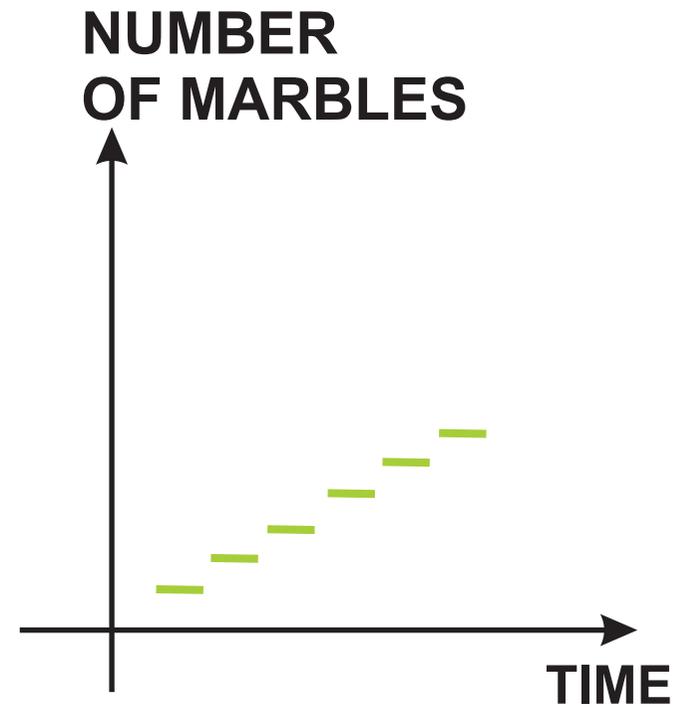
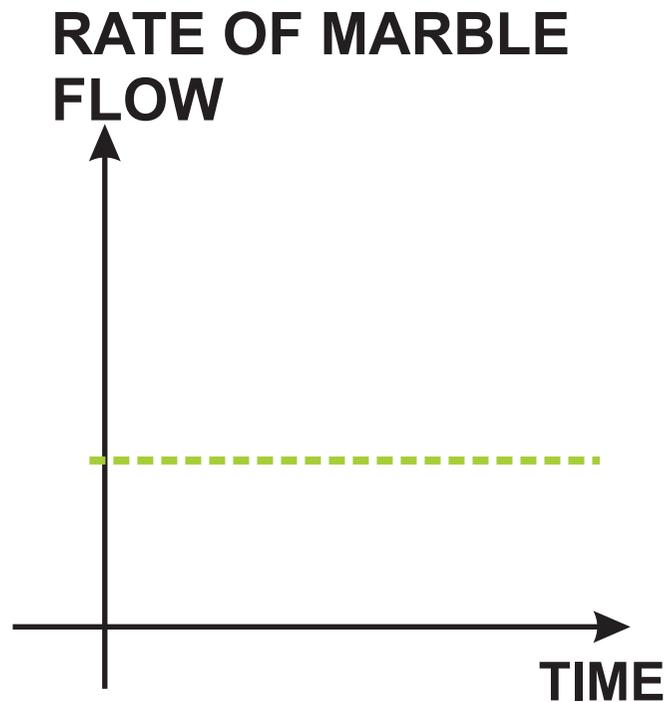


A MARBLE COUNTING CLOCK



**MARBLES FLOW
INTO THE BUCKET AT
A CONSTANT RATE.
NUMBER OF
MARBLES IN BUCKET
IS MEASURE OF TIME**

RELATION BETWEEN NUMBER OF MARBLES IN BUCKET AND RATE MARBLES FALLING



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NOTE: DISCRETE LEVELS ONLY

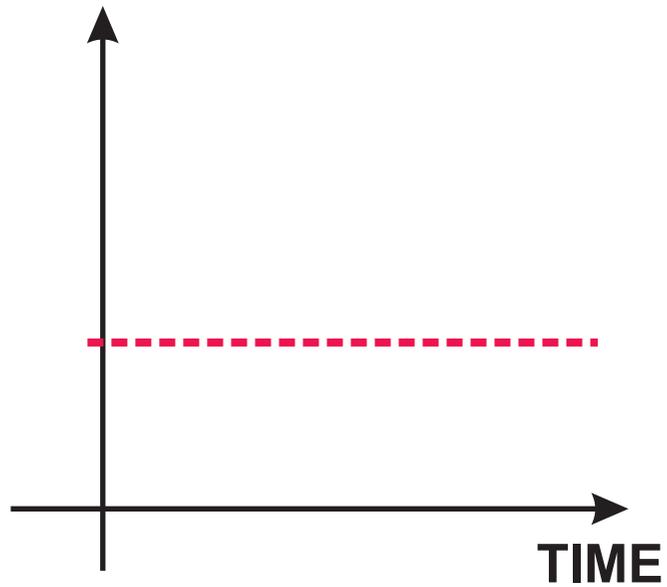
THE CAR CLOCK

**A CAR TRAVELS AT A CONSTANT
SPEED IN A STRAIGHT LINE
DISTANCE ON ROAD MEASURES TIME**

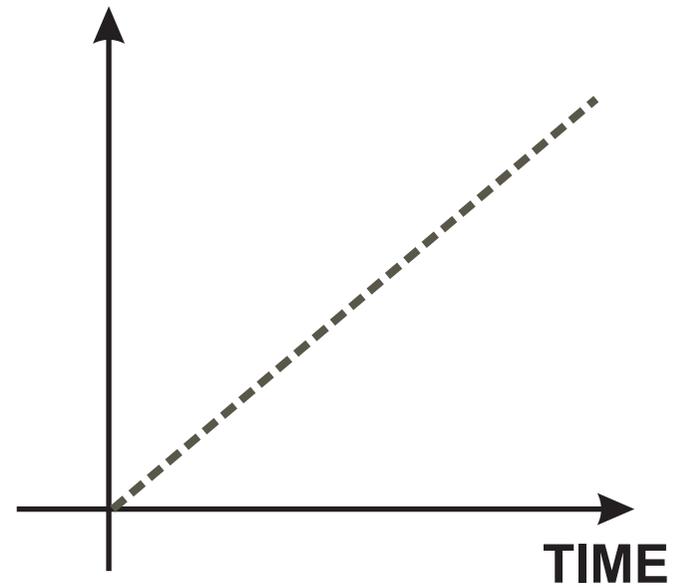


RELATION OF POSITION ON ROADWAY AND RATE OF SPEED

RATE OF SPEED



POSITION ON ROADWAY



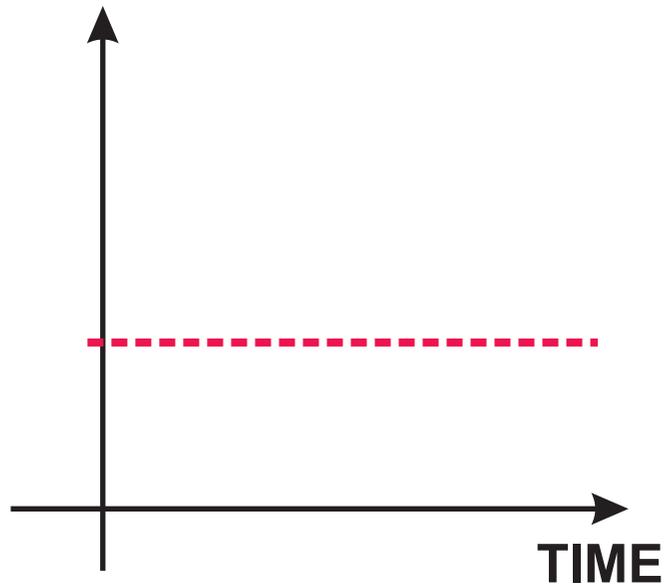
THE CAPACITOR CLOCK

**A CAPACITOR IS CHARGED BY A
CONSTANT CURRENT SOURCE, VOLTAGE
ON CAPACITOR IS A MEASURE OF TIME**

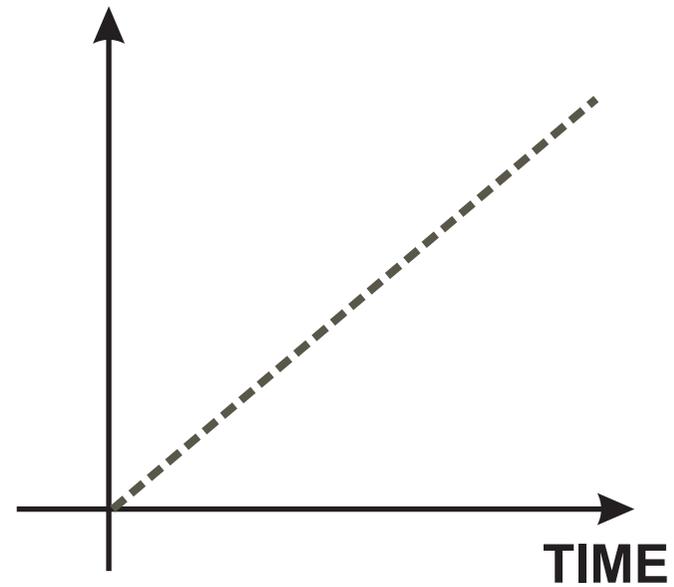


RELATION OF VOLTAGE ON CAPACITOR AND RATE OF CURRENT

RATE OF CURRENT



VOLTAGE ON CAPACITOR



DEFINITION:

CLOCK PHASE IS THAT QUANTITY THAT RESULTS WHEN A FLOW OR RATE IS ACCUMULATED OR SUMMED

EXPRESSED MATHEMATICALLY;

CLOCK PHASE = \int RATE , FOR ANALOG CLOCKS

CLOCK PHASE = \sum DISCRETE RATE, DISCRETE CLOCKS

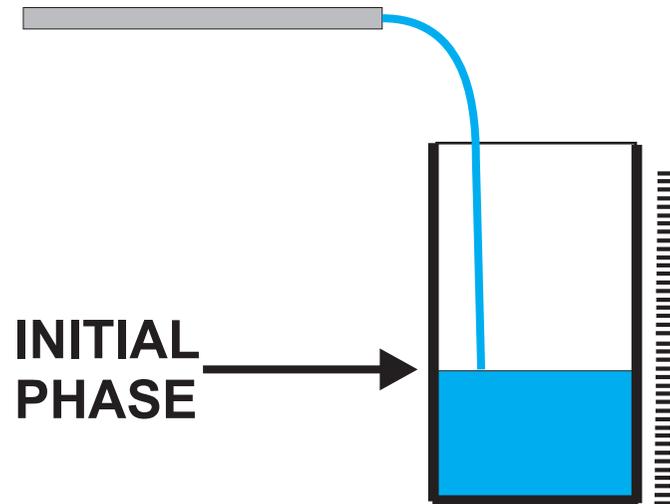
IF, AS WE HAVE ASSUMED THAT CLOCK RATE IS A CONSTANT THEN CLOCK PHASE IS A MEASURE OF TIME

INITIAL PHASE PROBLEM

WHEN ANY CLOCK IS OBSERVED IT CAN HAVE AN *INITIAL PHASE*.

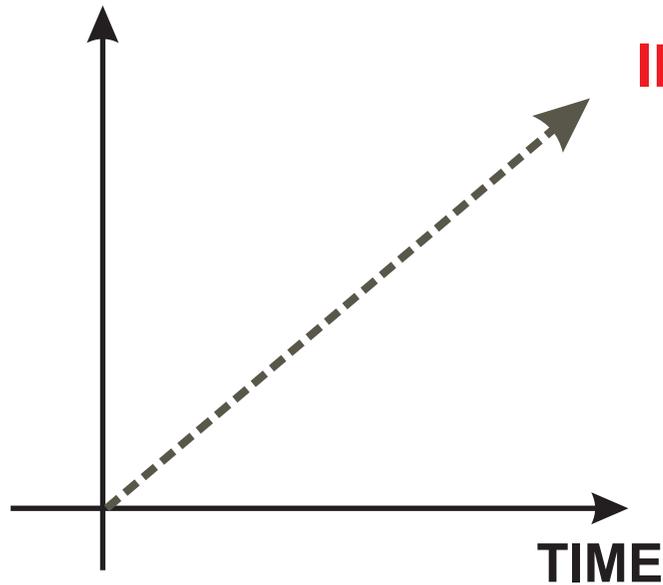
FOR EXAMPLE THERE MAY BE A LEVEL OF WATER IN THE BUCKET WHEN WE FIRST START OBSERVING THE SIMPLE WATER CLOCK.

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THE NEED FOR A CYCLE OF PHASE

CLOCK PHASE

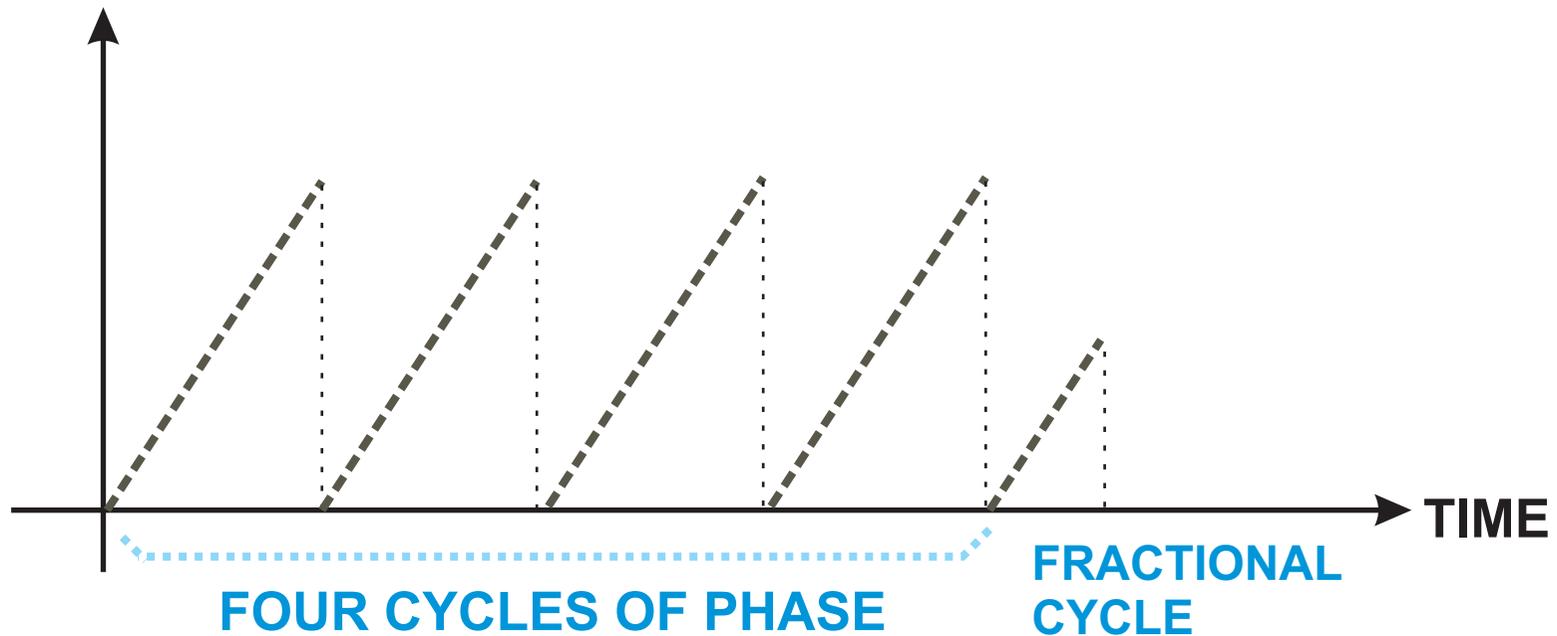


INCREASES TO INFINITY!

THE ACCUMULATION OF A CONSTANT RATE AD INFINITUM LEADS TO INFINITE PHASE!

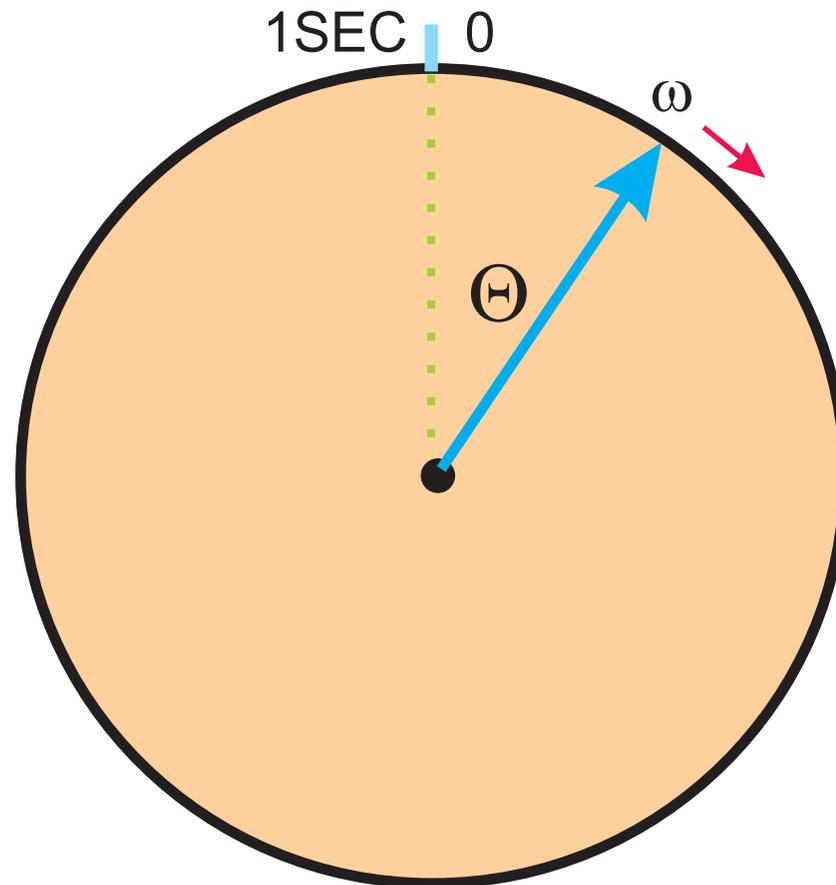
A CLOCK THAT RESETS ITS PHASE PERIODICALLY HAS *PHASE CYCLES*

CLOCK PHASE



TOTAL PHASE = 4 CYCLES + FRACTION OF CYCLE

ROTATIONAL CLOCKS SOLVE THE INFINITE PHASE PROBLEM BY RESETTING THE PHASE TO ZERO JUST BY THE NATURE OF SIMPLE ROTATION.



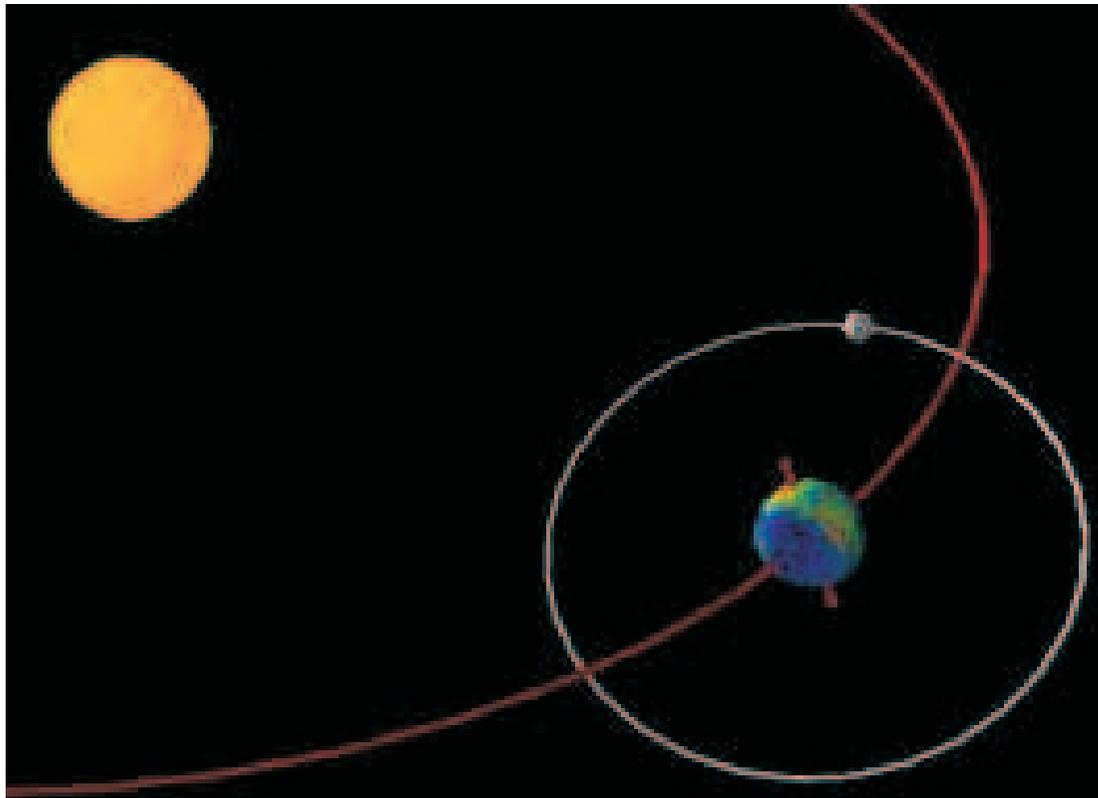
**ROTATIONAL CLOCKS ARE SOME OF THE OLDEST
CLOCKS KNOWN.**

**THE COMMON MECHANICAL STOP WATCH IS A MODERN
EXAMPLE**



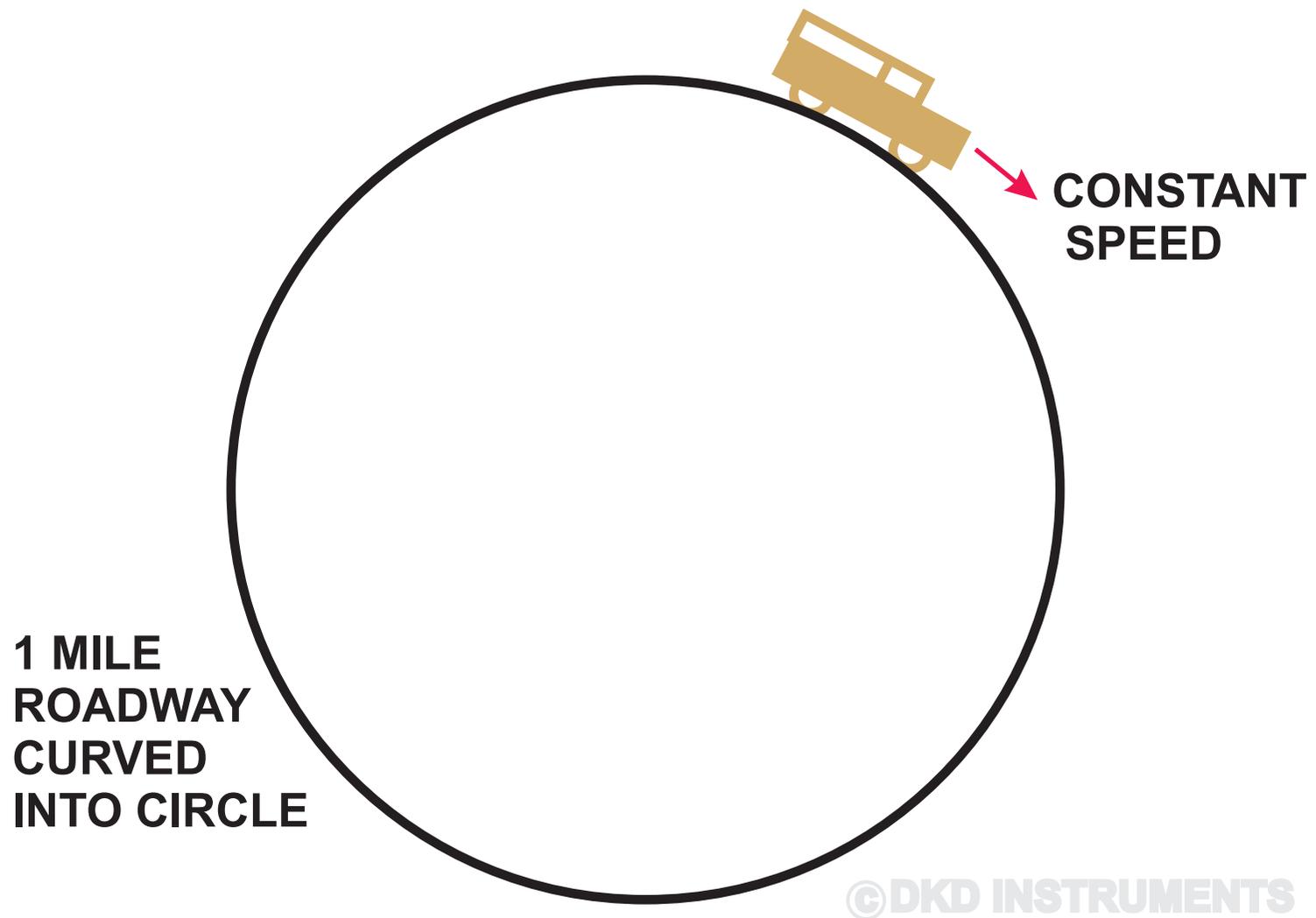
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THE EARTH'S ROTATION AROUND ITS AXIS, THE MOON'S ROTATION AROUND THE EARTH AND THE EARTH AROUND SUN HAVE BEEN USED SINCE ANCIENT TIMES AS A ROTARY TYPE CLOCK.

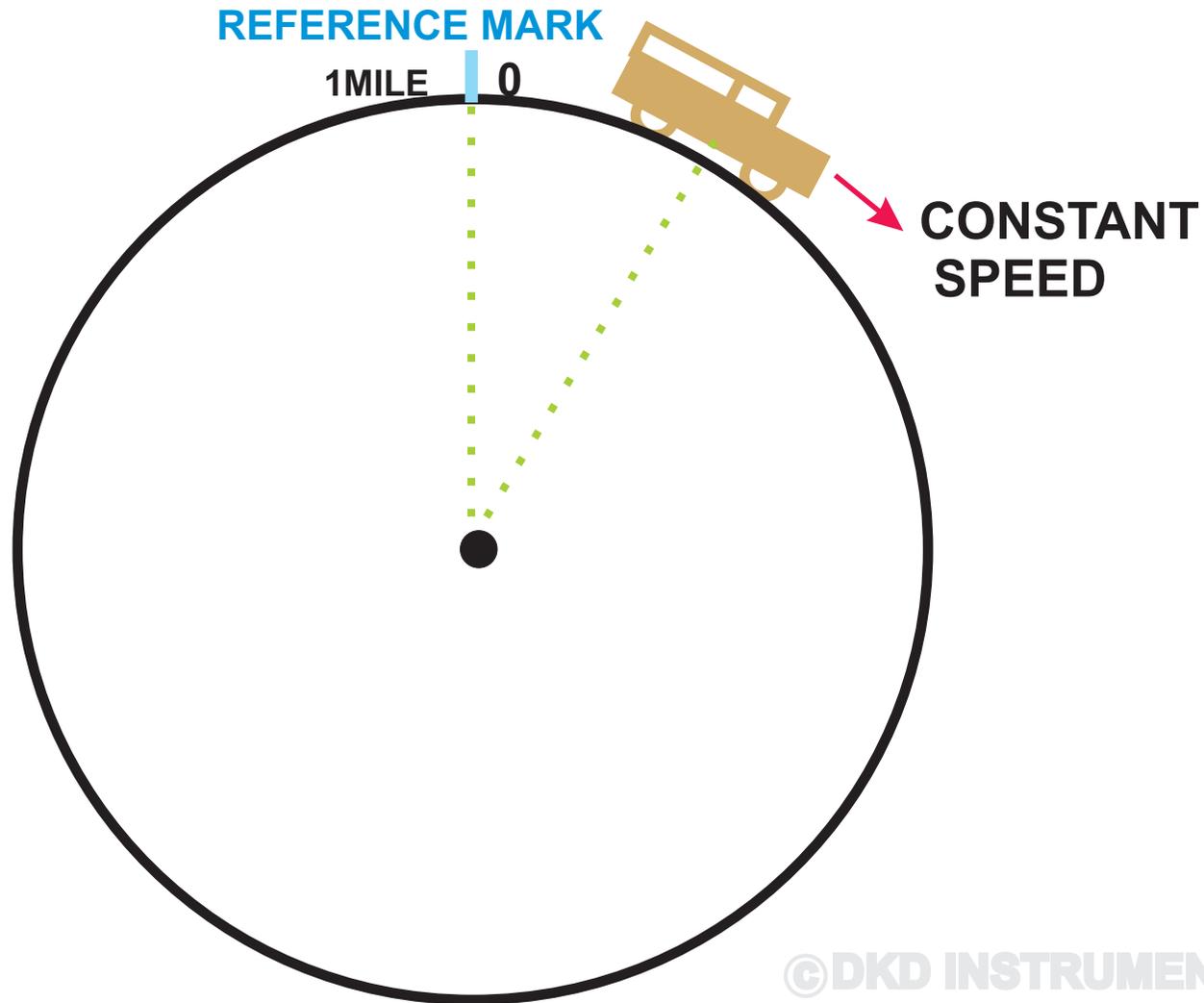


THE BASICS OF ROTATIONAL TYPE CLOCKS

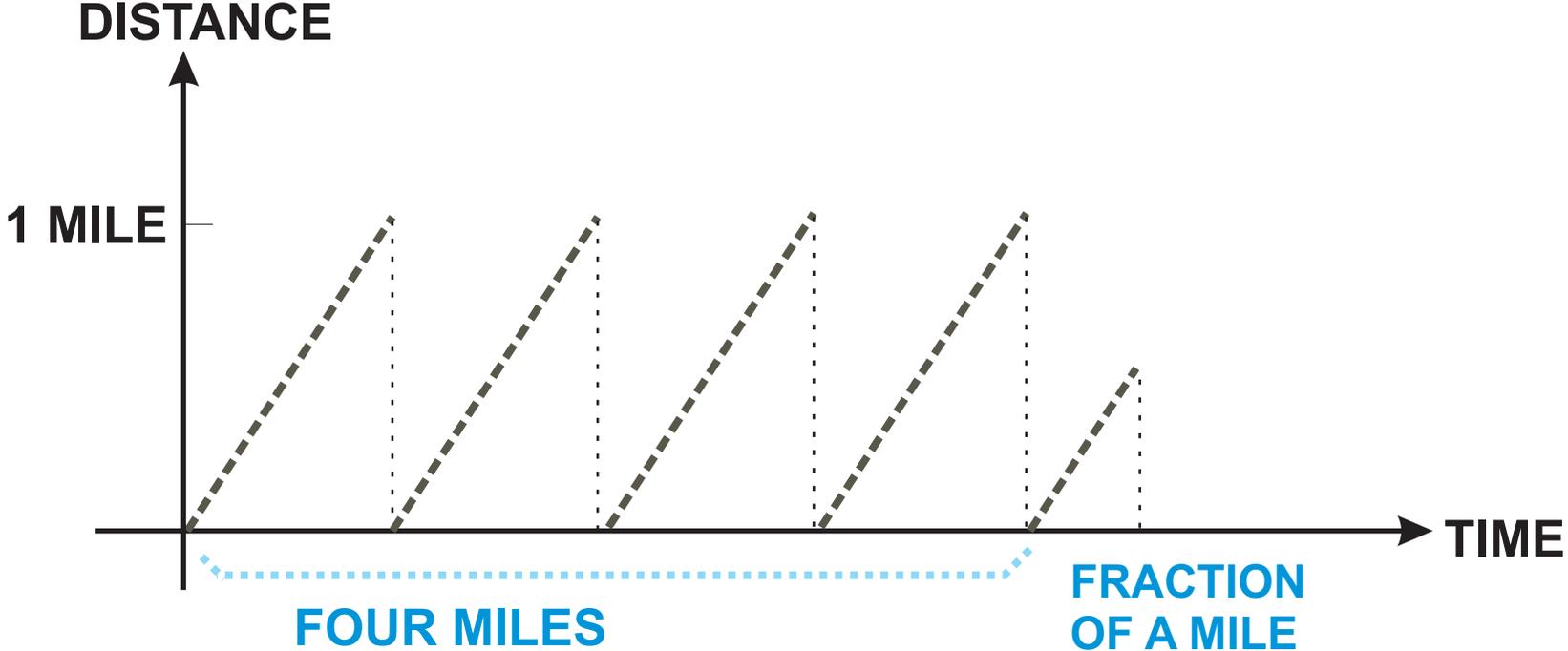
EXAMPLE::: CAR ON CIRCLE ROADWAY CLOCK



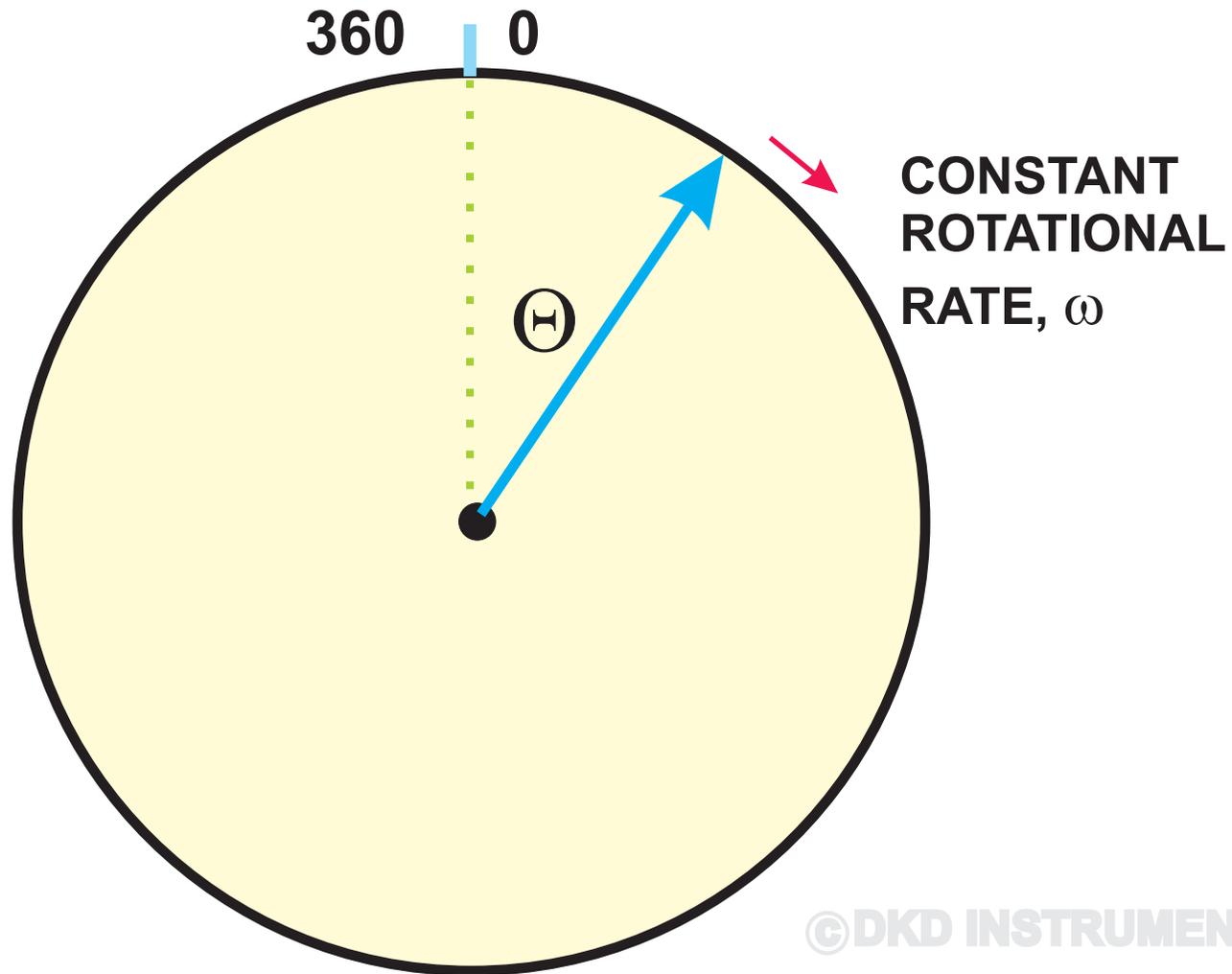
NOW MEASURE CAR POSITION FROM ARBITRARY REFERENCE MARK



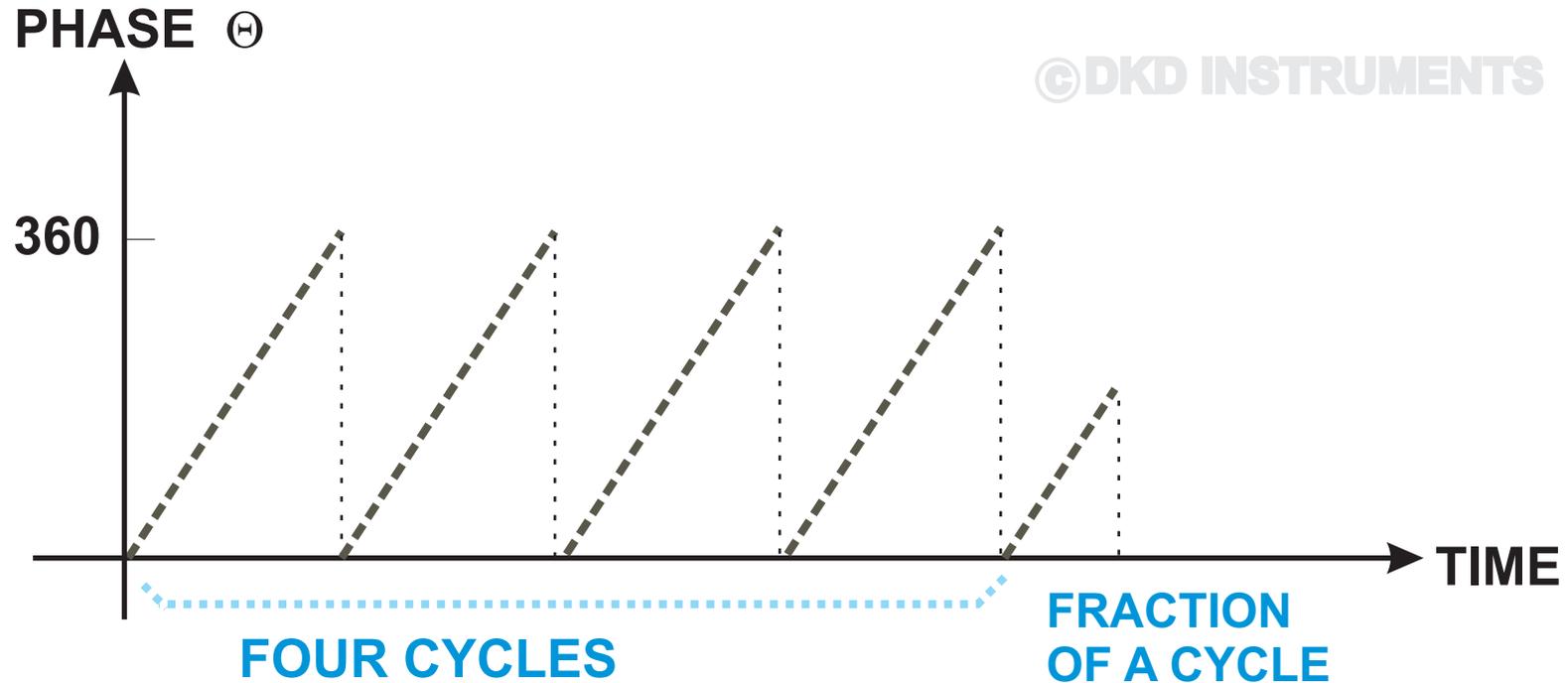
PLOT OF CAR POSITION SHOWS CLASSIC PHASE CYCLES



**NOW ABSTRACT OUR CAR MODEL TO CLASSIC
ROTARY CLOCK WITH RATE ω AND PHASE θ**



PLOT OF POINTER TIP EXPRESSED AS ANGLE Θ SHOWS CLASSIC PHASE CYCLES



where $\Theta = \int \omega dt + \text{Initial Angle}$

BASIC ROTARY CLOCK SUMMARY

INTEGRATION OF ROTATION RATE GIVES CHANGE IN POINTER PHASE

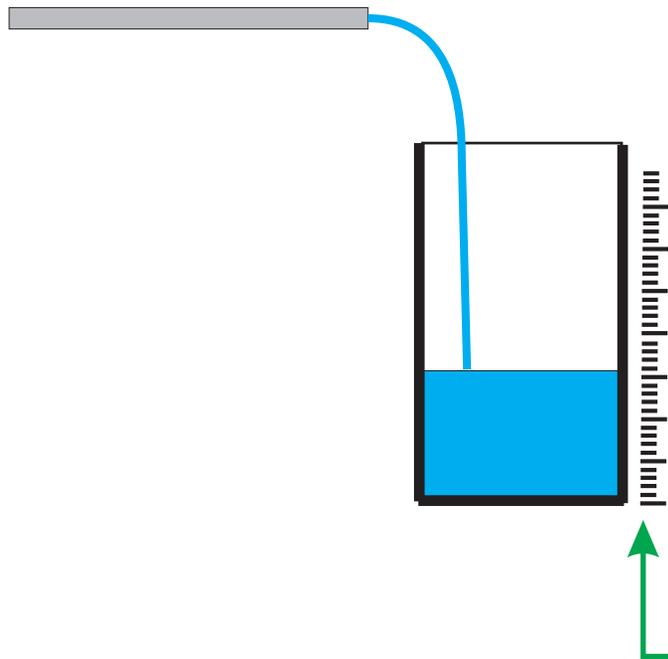
AUTOMATICALLY RESETS PHASE CREATING PHASE CYCLES

PHASE ANGLE Θ CAN BE MEASURED IN DEGREES, RADIANS, TIME (0 TO 1 SEC, ECT), OR ANY CONVENIENT UNIT.



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MEASURING CLOCK PHASE



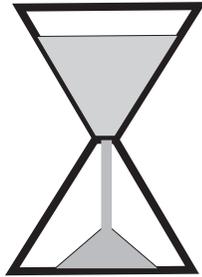
FOR THE WATER CLOCK WE
COULD MEASURE CLOCK PHASE
AS A LIQUID HEIGHT OR VOLUME

PHASE UNITS COULD BE INCHES,
METERS, LITERS, GALLONS ETC.

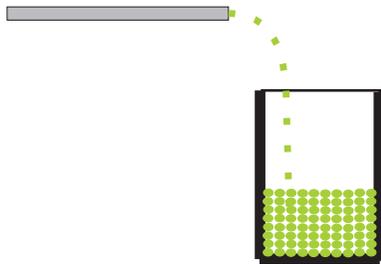
IF THE PHASE UNITS ARE
INSCRIBED ON THE CLOCK ITSELF
SUCH MARKS ARE CALLED
TIMING MARKS

VARIOUS CLOCKS AND UNITS OF PHASE

PHASE IN NATIVE UNITS



**INCHES, METERS, VOLUME
ETC.**



NUMBER OF MARBLES

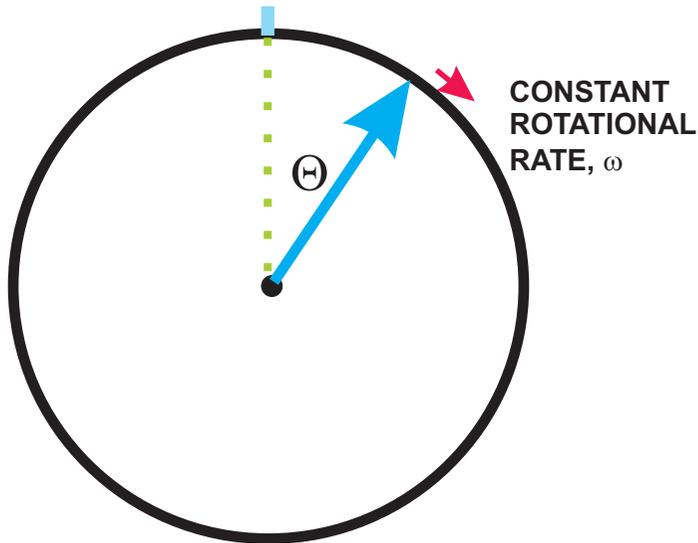


**METERS, KILOMETERS, MILES,
ECT.**



**VOLTS, STORED CHARGE IN
COULOMBS, ECT**

VARIOUS CLOCKS AND UNITS OF PHASE, CONT



ROTARY CLOCK PHASE CAN BE EXPRESSED IN CYCLES, RADIANS, DEGREES, SECONDS, MINUTES, HOURS, 1UNIT(0 to 1), etc...

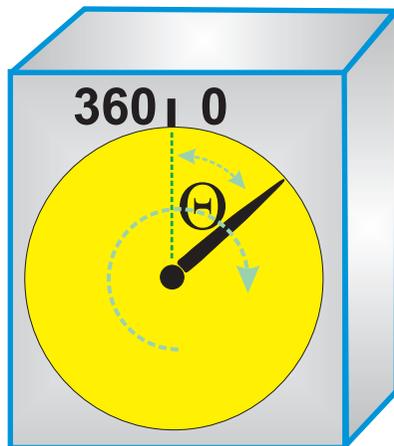
To relate native phase units to time in Seconds, divide by the clock rate expressed as native-phase per unit time, Or in general : $\Phi(\text{sec}) = \Theta / \omega$, Where: Θ is the phase expressed in Native Units, ω is the rate expressed in Native Units/sec)

BASIC PHASE MEASUREMENT

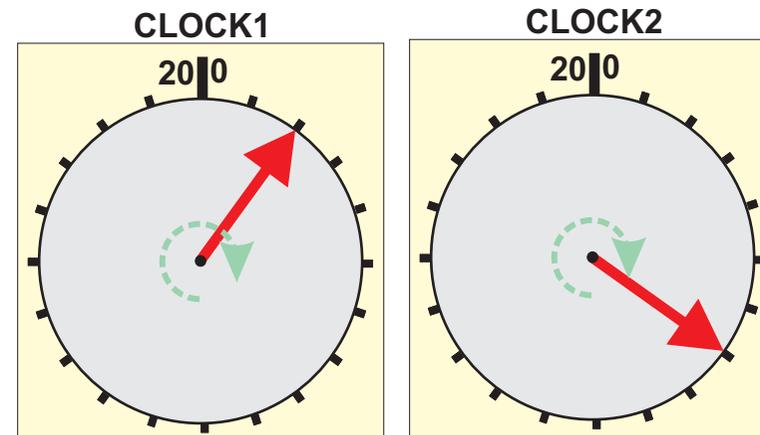
THERE ARE TWO FUNDAMENTAL TYPES OF PHASE MEASUREMENTS FOR A SINGLE INSTANT OF TIME:

1) CLOCK PHASE AS RECORDED AGAINST ITS OWN TIMING MARK

2) THE PHASE DIFFERENCE BETWEEN TWO IDENTICAL CLOCKS
PHASE AS RECORDED AT THE SAME INSTANT



PHASE wrt
TIMING MARK IS \oplus



PHASE DIFFERENCE IS 5 TICKS

SUMMARY OF PHASE MEASUREMENT

1) PHASE CAN HAVE MANY UNITS

2) PHASE IS USUALLY EXPRESSED RELATIVE TO A TIMING MARK

3) UNITS OF PHASE ARE OFTEN ARBITRARY.

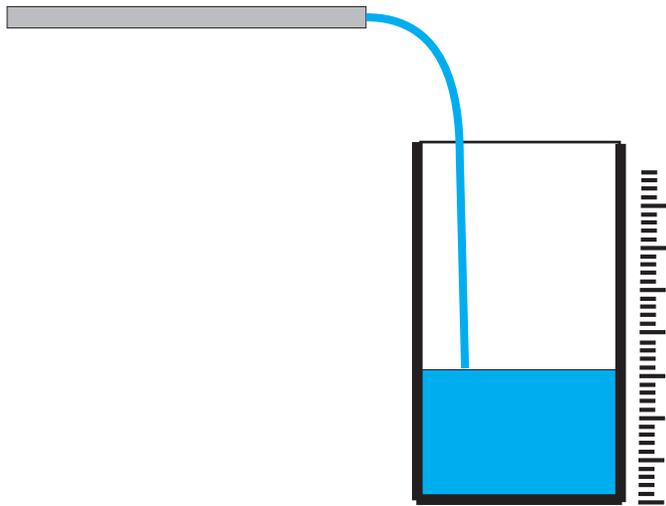
4) THE TERM "ZERO PHASE" IS MEANINGLESS WITHOUT CLEARLY STATING THE REFERENCE USED.

5) THE UNIT OF TIME WE CALL THE SECOND IS AN INTERNATIONALLY AGREED UPON UNIT OF TIME , OR PHASE



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MEASURING CLOCK RATE



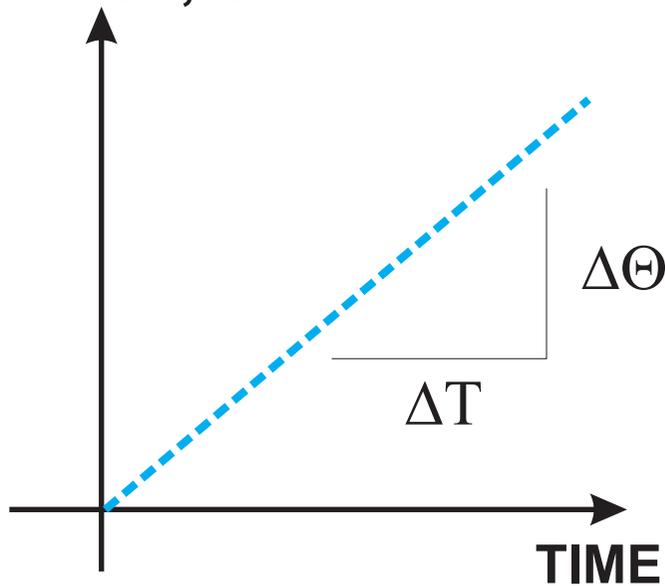
FOR THE WATER CLOCK WE COULD MEASURE CLOCK RATE BY RECORDING WATER LEVEL(PHASE) AT TWO DIFFERENT TIMES.

IF THE TIME INTERVAL IS KNOWN BETWEEN THESE TWO PHASE MEASUREMENTS THEN

$$\text{CLOCK RATE} = \Delta\Theta/\Delta T$$

MEASURING CLOCK RATE BY SLOPE

WATER CLOCK
PHASE, Θ



$$\text{CLOCK RATE} = \Delta \Theta / \Delta T$$

THE FUNDAMENTAL PROBLEM OF MEASURING CLOCK RATE

**TO MEASURE A CLOCKS RATE REQUIRES THE
PRESENCE OR USE OF ANOTHER CLOCK!**

**THIS OTHER CLOCK APPEARS AS THE ΔT TERM IN THE
FORMULA $\Delta\Theta/\Delta T$**

**A CLOCK USED TO MEASURE THE RATE OF ANOTHER
CLOCK IS OFTEN CALLED A REFERENCE CLOCK OR A
STANDARD CLOCK**

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MEASURING CLOCK RATE SUMMARY

- ▶ **CLOCK RATE MEASUREMENT REQUIRES A MINIMUM OF TWO PHASE MEASUREMENTS OF THAT CLOCK SEPARATED BY A KNOWN AMOUNT OF TIME.**
- ▶ **CLOCK RATE MEASUREMENT IS THEREFORE A DIFFERENTIAL PHASE MEASUREMENT.**
- ▶ **RATE MEASUREMENT IS AT ITS HEART A PHASE COMPARISON PROCESS BETWEEN THE CLOCK BEING INVESTIGATED AND THE STANDARD OR REFERENCE CLOCK.**
- ▶ **A CLOCK WITH ZERO RATE HAS A PHYSICAL MEANING. SOMETHING HAS STOPPED ROTATING, FLOWING, ETC.**
- ▶ **CLOCK RATE CAN BE DETERMINED FROM CLOCK PHASE MEASUREMENTS, THE CONVERSE IS NOT TRUE DUE TO (TYPICALLY) UNKNOWN INITIAL PHASE**

CYCLE SLIPPING IN ROTARY CLOCKS

IF WE OBSERVE TWO IDENTICAL CLOCKS LONG ENOUGH EVENTUALLY WE WILL OBSERVE WHAT IS CALLED A *CYCLE SLIP*

THIS IS TRUE BECAUSE IT IS IMPOSSIBLE FOR TWO CLOCKS TO RUN AT EXACTLY THE SAME RATE.

BELOW ARE SOME SNAP SHOTS (TAKEN WHEN CLOCK2 HITS ZERO) OF TWO OF THE SAME TYPE OF CLOCKS. CLOCK1 IS FAST WITH RESPECT TO CLOCK2, OUR REFERENCE CLOCK.

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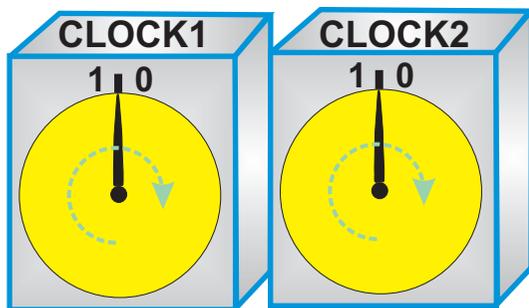


PHOTO 1

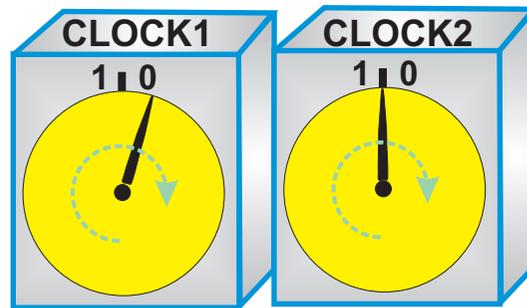


PHOTO 2

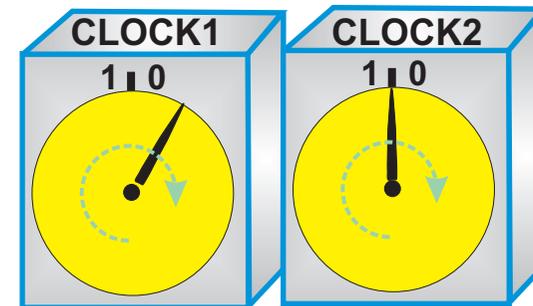


PHOTO 3

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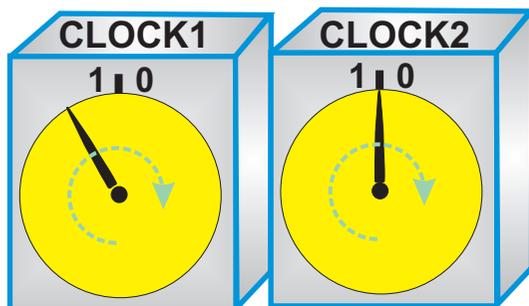


PHOTO 33

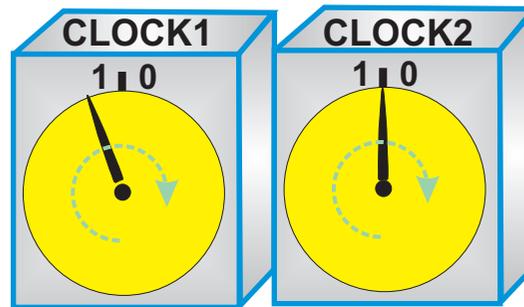


PHOTO 35

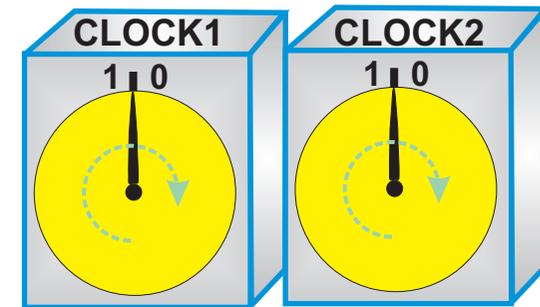


PHOTO 36

CYCLE SLIPPING IN ROTARY CLOCKS(CONT)

IN PHOTO1 CLOCK1 AND CLOCK2 READ EXACTLY THE SAME TIME.

AFTER A FULL CYCLE OF CLOCK2, CLOCK1 IS NOW SLIGHTLY AHEAD OF CLOCK2 DUE TO THE SLIGHTLY HIGHER RATE OF CLOCK2 AS SHOWN IN PHOTO2.

BY PHOTO36 THE TWO CLOCKS ARE READING THE SAME TIME AGAIN. IF WE HAD COUNTED THE FULL CYCLES OF THE TWO CLOCKS WE WOULD SEE THAT CLOCK1 IS NOW AHEAD OF CLOCK2 BY A FULL CYCLE.

THIS IS WHAT IS CALLED A CYCLE SLIP.

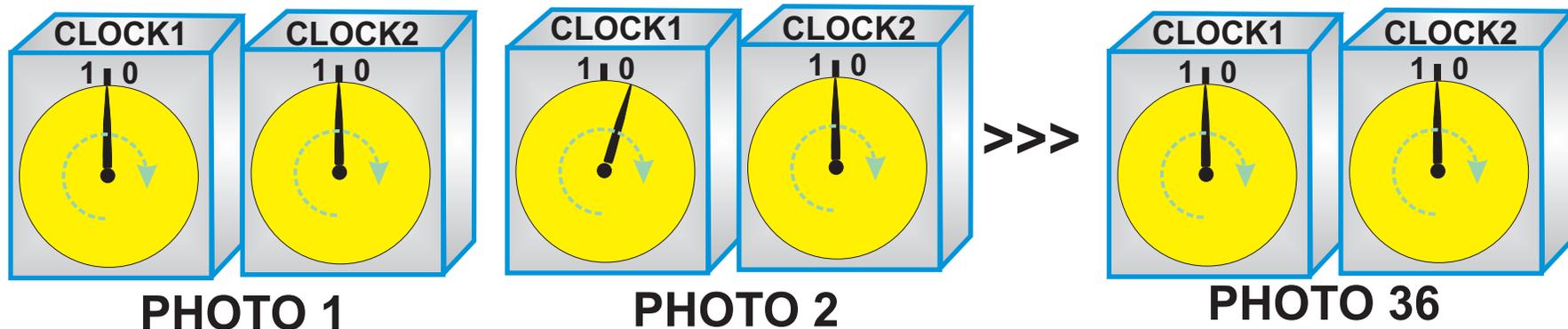
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THE TERM CYCLE SLIP REFERS TO A LOSS OR GAIN OF ONE OR MORE CYCLES DEVELOPING BETWEEN TWO ROTARY TYPE OR CYCLIC TYPE CLOCKS.

CYCLE SLIPS OCCUR BECAUSE OF RATE DIFFERENCES BETWEEN CLOCKS.

CYCLE SLIPS OCCUR OFTEN IN PHASE LOCK TYPE CONTROL LOOPS DUE TO DISTURBANCES THAT DISRUPT LOCK AND ALLOW UNCORRECTED RATE ERRORS TO DEVELOP.

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FUNDAMENTALS OF CLOCKS SUMMARY

- ▶ THE PRINCIPLE OF OPERATION OF ALL CLOCKS IS THE SAME:: THEY ACCUMULATE OR INTEGRATE A CONSTANT RATE WHICH RESULTS IN A MEASUREMENT OF TIME.
- ▶ THE ACCUMULATED RATE IS CALLED PHASE
- ▶ PHASE AND RATE CAN HAVE MANY UNITS DEPENDING ON THE CLOCK THEY ARE BASED ON (Native Units)
- ▶ THE TWO BASIC PHASE MEASUREMENTS ARE: PHASE WITH RESPECT TO A TIMING MARK AND PHASE DIFFERENCE BETWEEN TWO CLOCKS.
- ▶ TO CONVERT A PHASE THAT IS EXPRESSED IN NATIVE UNITS INTO TIME IN SECONDS DIVIDE BY THE NATIVE RATE PER SECOND.
- ▶ SOME CLOCKS ACCUMULATE PHASE IN AN ANALOG FASHION OTHERS IN DISCRETE STEPS
- ▶ THE MEASUREMENT OF CLOCK RATE IS A PHASE COMPARISON PROCESS BETWEEN TWO CLOCKS, TYPICALLY A STANDARD OR REFERENCE CLOCK AND THE CLOCK BEING MEASURED.
- ▶ ROTARY CLOCKS RESET THEIR PHASE BACK TO ZERO EVERY CYCLE. THEY ARE OFTEN MULTI-DIAL ARRANGEMENTS WITH INTEGER CYCLE COUNT RELATIONSHIPS BETWEEN DIALS.
- ▶ CYCLE SLIPS CAN OCCUR BETWEEN TWO OR MORE CYCLE COUNTING TYPE CLOCKS. THEY ARISE FROM RATE DIFFERENCES BETWEEN THE CLOCKS.

APPENDIX II

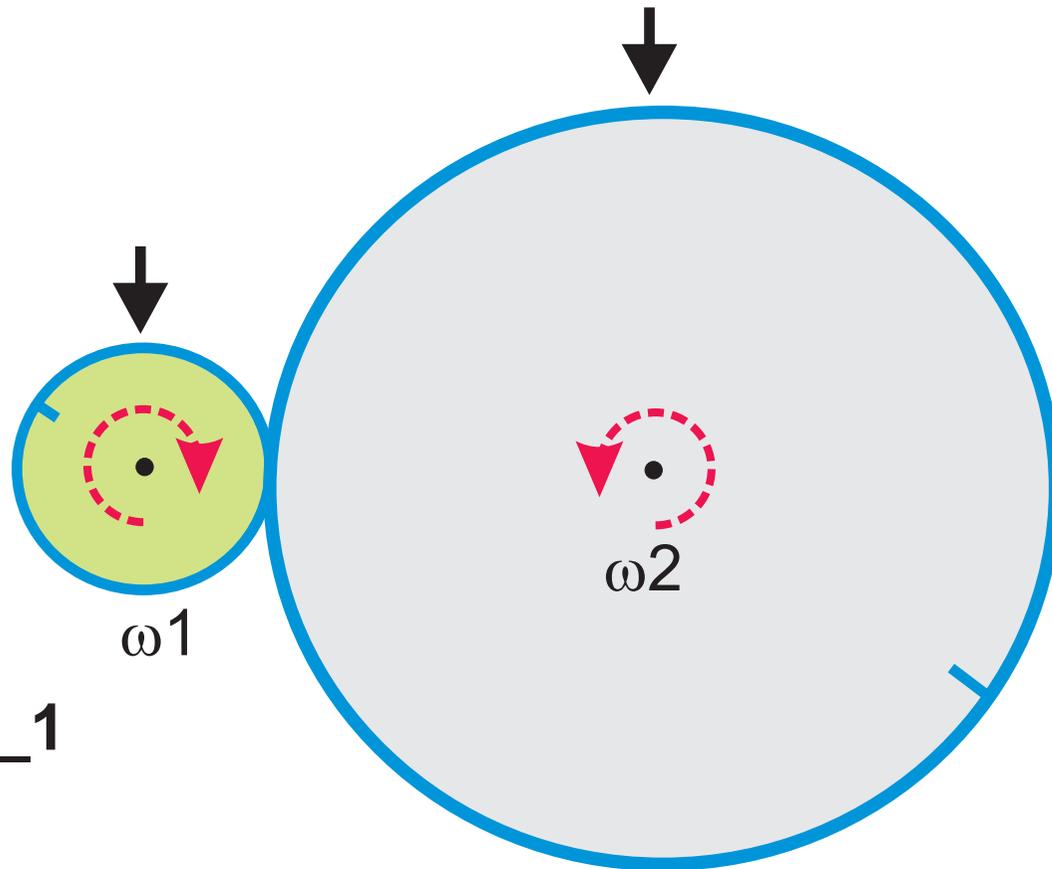
MECHANICAL MULTI-DIAL ROTARY CLOCKS AND A MULTI-DIAL MODEL GPS CLOCK



MECHANICAL MULTI-DIAL ROTARY CLOCKS

BASIC PHASE AND RATE MULTIPLICATION/DIVISION BETWEEN DIALS OF DIFFERENT DIAMETERS

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$$\text{RATIO} = \text{DIA_2} / \text{DIA_1}$$

$$\omega_2 = \omega_1 / \text{RATIO}$$

$$\Theta_2 = \Theta_1 / \text{RATIO}$$

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IF RATIO IS AN INTEGER THEN
LARGE DIAL IS COUNTING FULL
CYCLES OF SMALL DIAL

ROTARY ANALOG DIALS AND DIGITAL DIALS

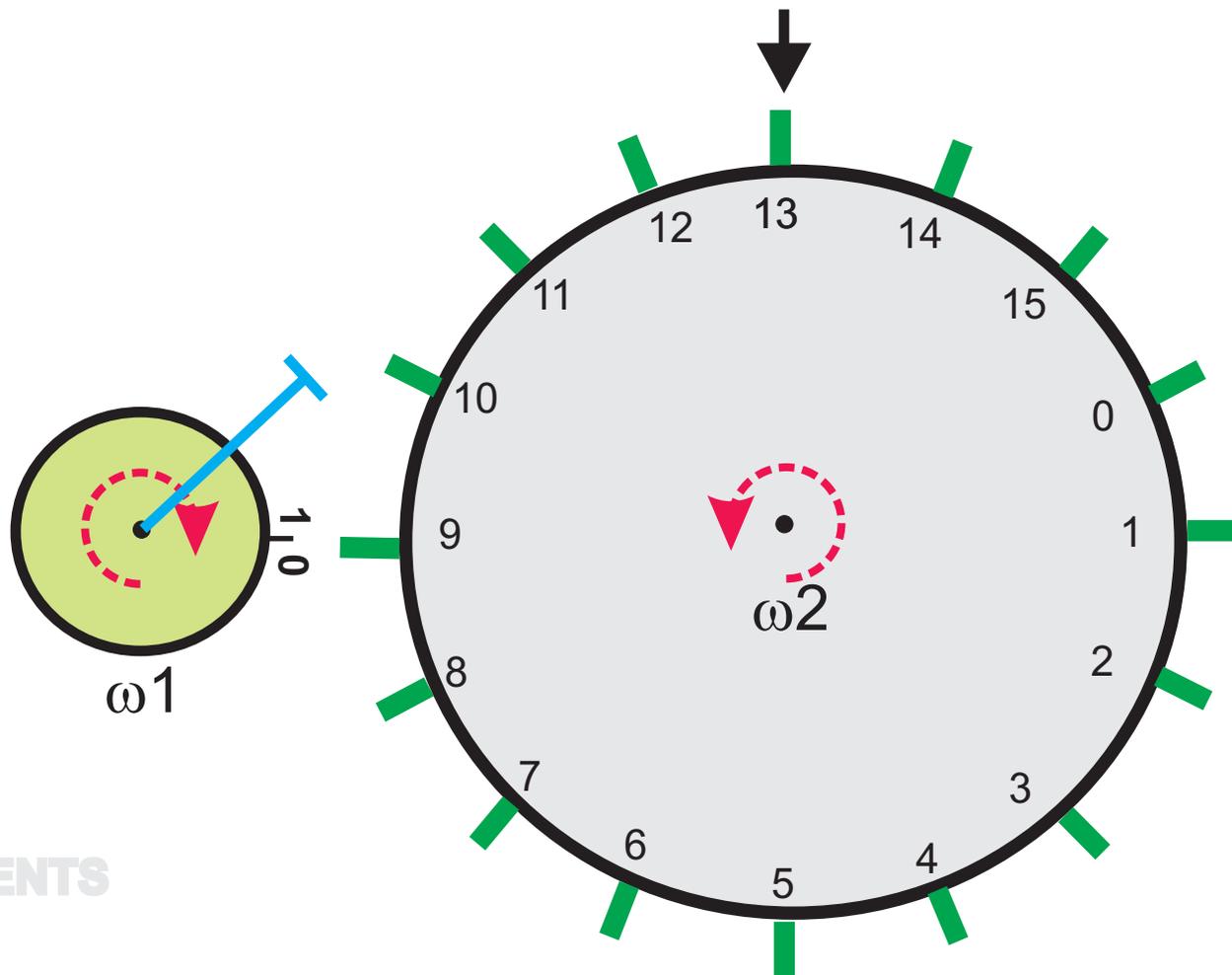
THE SMALL DIAL CAN HAVE ANY PHASE FROM ZERO TO 1 CYCLE. IT IS CALLED AN ANALOG DIAL. THE LARGE DIAL COUNTS FULL ROTATIONS OF THE SMALL DIAL . ITS PHASE IS DISCRETE. THE LARGE DIAL IS A *DIGITAL* DIAL AND AS SHOWN IN THIS EXAMPLE HAS 16 PHASE STATES. THE SMALL DIAL SHOWS A PHASE OF 3/4 CYCLE WHILE THE LARGE DIAL SHOWS A PHASE OF 13 CYCLES OF THE SMALL DIAL.

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BY DEFINITION A DIGITAL DIAL HAS AN INTEGER RELATION OF ITS RATE TO THE RATE OF DIAL DRIVING IT;

$$\omega_1 = N \cdot \omega_2$$

where N is an Integer



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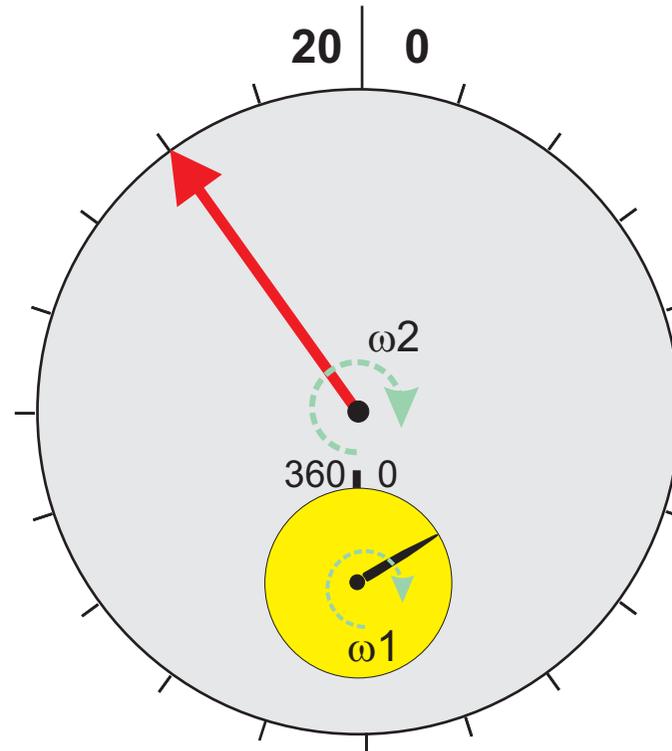
SCHEMATIC REPRESENTATION OF A 20 to 1, TWO DIAL CLOCK
with DIGITAL and ANALOG DIALS

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HERE THE INNER DIAL IS A
ANALOG DIAL, ALL PHASES ARE
POSSIBLE.

THE OUTER DIAL ONLY
INCREASES BY ONE TICK WHEN
INNER DIAL COMPLETES 1 CYCLE.

OUTSIDE DIAL COUNTS A
MAXIMUM OF 20CYCLES OF INNER
DIAL AND THEN STARTS AGAIN



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$$\omega_2 = \omega_1 / 20$$

PHASE AS SHOWN WOULD BE = 18cycles + 70DEG.

SYNTONIZED CLOCK DIALS

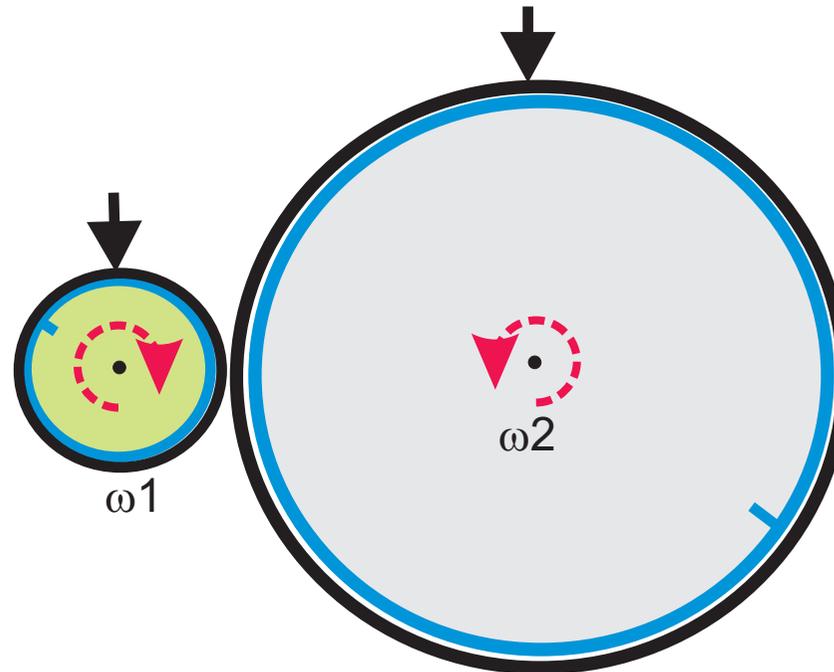
A SPECIAL SET OF CLOCK DIALS CAN BE CONSTRUCTED THAT KEEP THEIR RATE RELATIONSHIP ~INTACT BUT ALLOW THEIR PHASE RELATIONSHIP TO DRIFT.

THIS CAN BE MODELED AS TWO CLOCK DIALS WITH RUBBER EDGES JUST TOUCHING (BELT DRIVEN DIALS ARE ANOTHER EXAMPLE). IN SUCH A CONSTRUCTION OCCASIONAL PHASE SLIPS OCCUR BUT ON AVERAGE THE RATES MAINTAIN A FIXED RELATION.

WHEN TWO OR MORE CLOCK DIAL RATES ARE HELD STEADY IN *RATE ONLY* IT IS CALLED *SYNTONIZED*.

USUALLY THE TERM SYNTONIZED IS USED TO DESCRIBE TWO SEPARATE CLOCKS RUNNING AT THE SAME RATE. HERE WE USE IT FOR TWO DIALS IN THE SAME CLOCK.

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$$\omega_2 \sim \omega_1 / \text{RATIO}$$

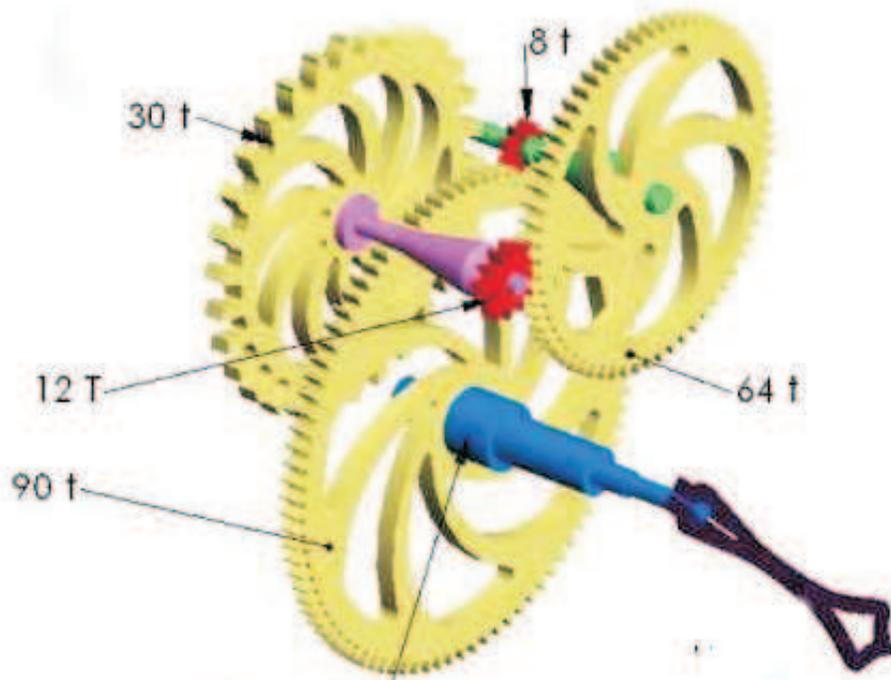
Θ_2 to Θ_1 relation is undefined

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SYNCHRONOUS CLOCK DIALS

THE CASE WHERE TWO OR MORE CLOCK DIALS REMAIN IN A FIXED *PHASE* AND *RATE* RELATIONSHIP IS CALLED *SYNCHRONOUS*. ALL GEAR CONNECTED TYPE CLOCKS ARE SYNCHRONOUS.

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$$\omega_n = \omega_m / \text{GEAR RATIO}$$

$$\Theta_n = \Theta_m / \text{GEAR RATIO}$$

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THE FOLLOWING SLIDE SHOWS A MULTI-DIAL CLOCK MODELED AFTER THE GPS TIMING SIGNALS.

OUR GPS MODEL CLOCK(FOR USE INSIDE A RECEIVER) IS MADE OF ANALOG AND DIGITAL DIALS. THE TWO ANALOG DIALS ARE THE C/A CHIP DIAL AND THE CARRIER PHASE DIAL. ALL OTHER DIALS ARE DIGITAL

THERE ARE;

**1540 CYCLES OF THE CARRIER PHASE DIAL FOR ONE CYCLE OF THE C/A CHIP DIAL
(FOR RECEIVERS THIS IS TRUE ONLY IN AN AVERAGE SENSE AS THESE TWO DIALS ARE USUALLY HELD IN SYNTONY ONLY)**

1023 CYCLES OF THE C/A CHIP DIAL FOR ONE CYCLE OF THE C/A CODE DIAL

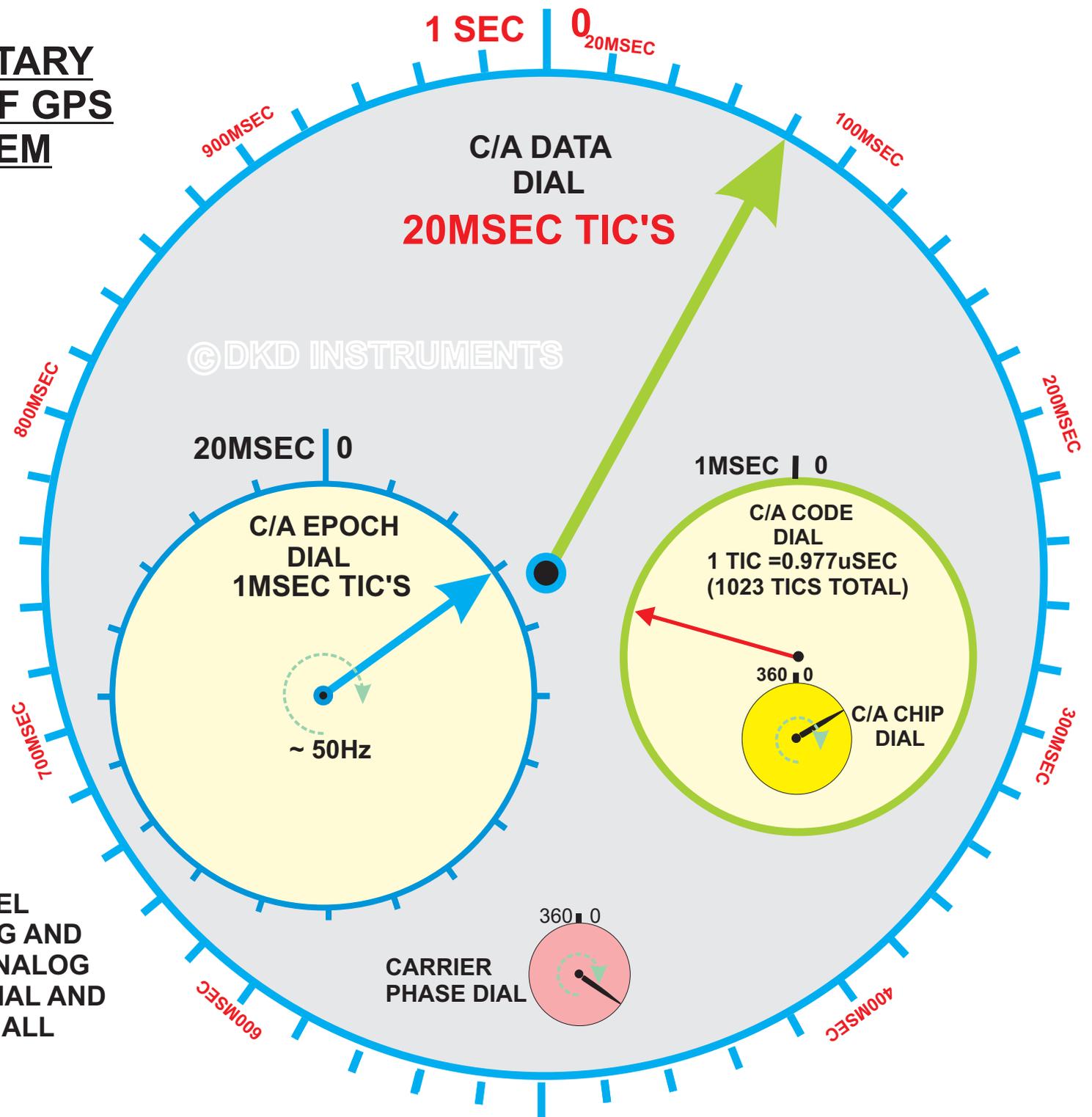
20 CYCLES OF THE C/A CODE DIAL FOR ONE CYCLE OF THE C/A EPOCH DIAL

50 CYCLES OF THE C/A EPOCH FOR ONE CYCLE OF THE 1 SECOND DIAL

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A MULTI-DIAL ROTARY CLOCK MODEL OF GPS C/A CLOCK SYSTEM



NOTE THAT OUR GPS MODEL CLOCK IS MADE OF ANALOG AND DIGITAL DIALS. THE TWO ANALOG DIALS ARE THE C/A CHIP DIAL AND THE CARRIER PHASE DIAL. ALL OTHER DIALS ARE DIGITAL

CLOCK DIALS with COMMENSURATE and NON COMMENSURATE RATES

TWO CLOCK DIALS ARE SHOWN BELOW. THE SMALLER DIAL, ω_1 , SPINS AT A FASTER RATE THEN THE LARGER DIAL, ω_2 .

THE ω_1 DIAL HAS A STROBE at ZERO TIMING MARK THAT CAPTURES THE PHASE STATE OF ω_2 DIAL.

IF BOTH DIALS ARE STARTED WITH THIER POINTERS AT ZERO TIMING MARKS WHAT WILL THE SEQUENCE OF SNAP SHOTS TAKEN BY THE STROBE REVEAL?

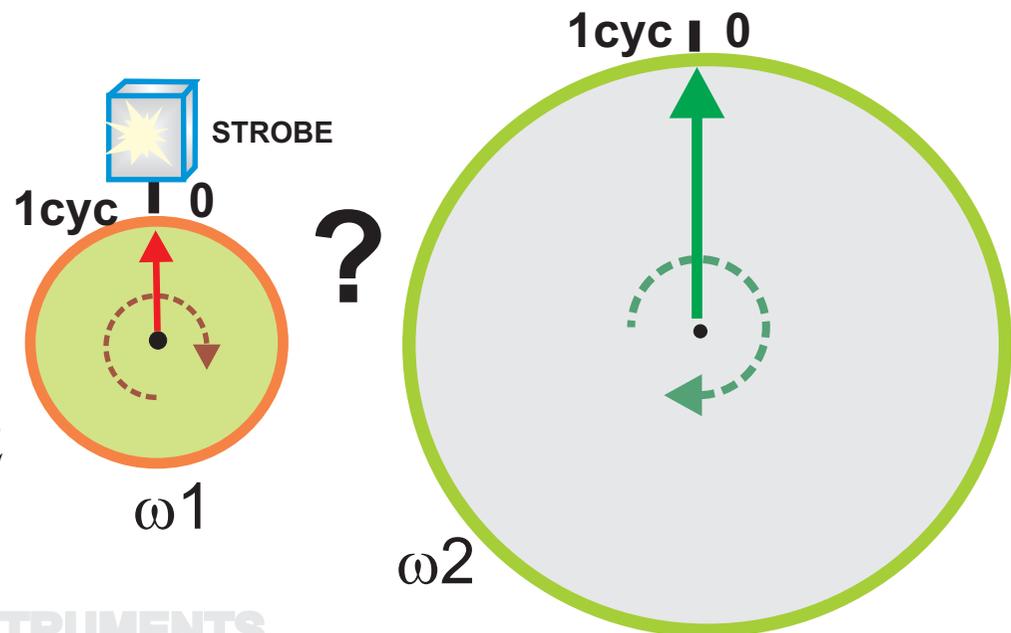
IN PARTICULAR, WILL THE TWO POINTERS EVER BE AT THEIR ZERO TIMING MARK AT THE SAME INSTANT AGAIN?(i.e. Commensurate)

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$$\omega_2 = \omega_1 / \text{DIAL_RATIO}$$

$$\text{DIAL_RATIO} = \omega_1 / \omega_2$$

$$\text{DIAL_RATIO} > 1$$



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CLOCK DIALS with COMMENSURATE and NON COMMENSURATE RATES cont.

THE ANSWER TO THE COMMENSURATE QUESTION JUST PROPOSED LIES IN THE TYPE OF NUMBER THE *DIAL_RATIO* IS:

CASE1: IF THE *DIAL_RATIO* IS A SIMPLE INTEGER , N, THEN EVERY N CYCLES OF THE ω_1 DIAL THE TWO DIALS WILL AGAIN BE COMMENSURATE. *The two dials in this case are said to be Commensurate.*

CASE2: IF THE *DIAL_RATIO* IS A RATIO OF TWO SIMPLE INTEGERS , N1/N2, THEN EVERY N_{factor} CYCLES OF ω_1 DIAL THE TWO DIALS WILL AGAIN BE COMMENSURATE, WHERE $N_{\text{factor}} = \text{CommonFactorsRemovedFromProductOf}[N1*N2]$. *If N_{factor} is small the two dials would be ~Commensurate. If N_{factor} is LARGE the two dials would be ~NON Commensurate. (see example below for N_{factor} Calculation)*

CASE3: IF THE *DIAL_RATIO* IS AN IRRATIONAL NUMBER THE TWO DIALS WILL NEVER AGAIN BE COMMENSURATE. *The two dials in this case are said to be Non- Commensurate.*

Let $\omega_1 = 357\text{Hz}$, Let $\omega_2 = 255\text{Hz}$
or $N_1 = 357$ and $N_2 = 255$

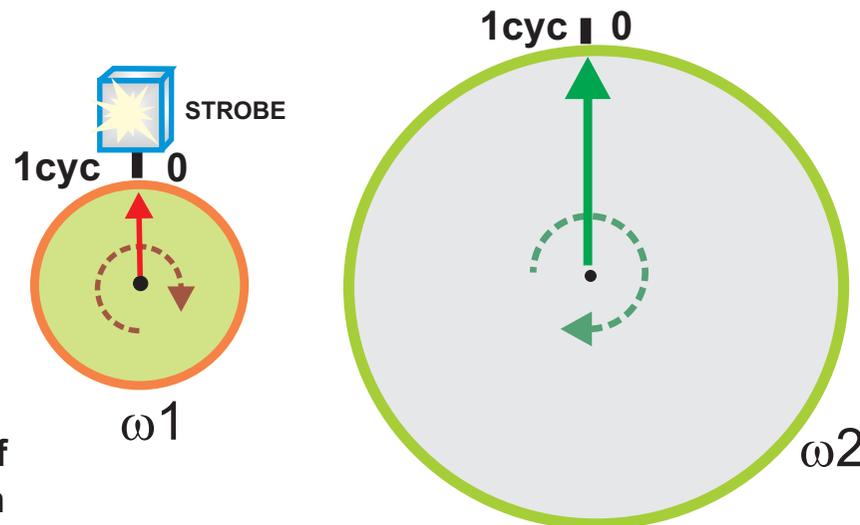
then $\text{Dial_Ratio} = N_1/N_2 \Rightarrow 1.4$

Factors of $N_1 = 7*17*3$

Factors of $N_2 = 5*17*3$

$N_{\text{factor}} = 7*5 \Rightarrow 35$

For every 35 cycles of the ω_1 dial, 25 cycles of the ω_2 dial occur and the two dials are again commensurate.

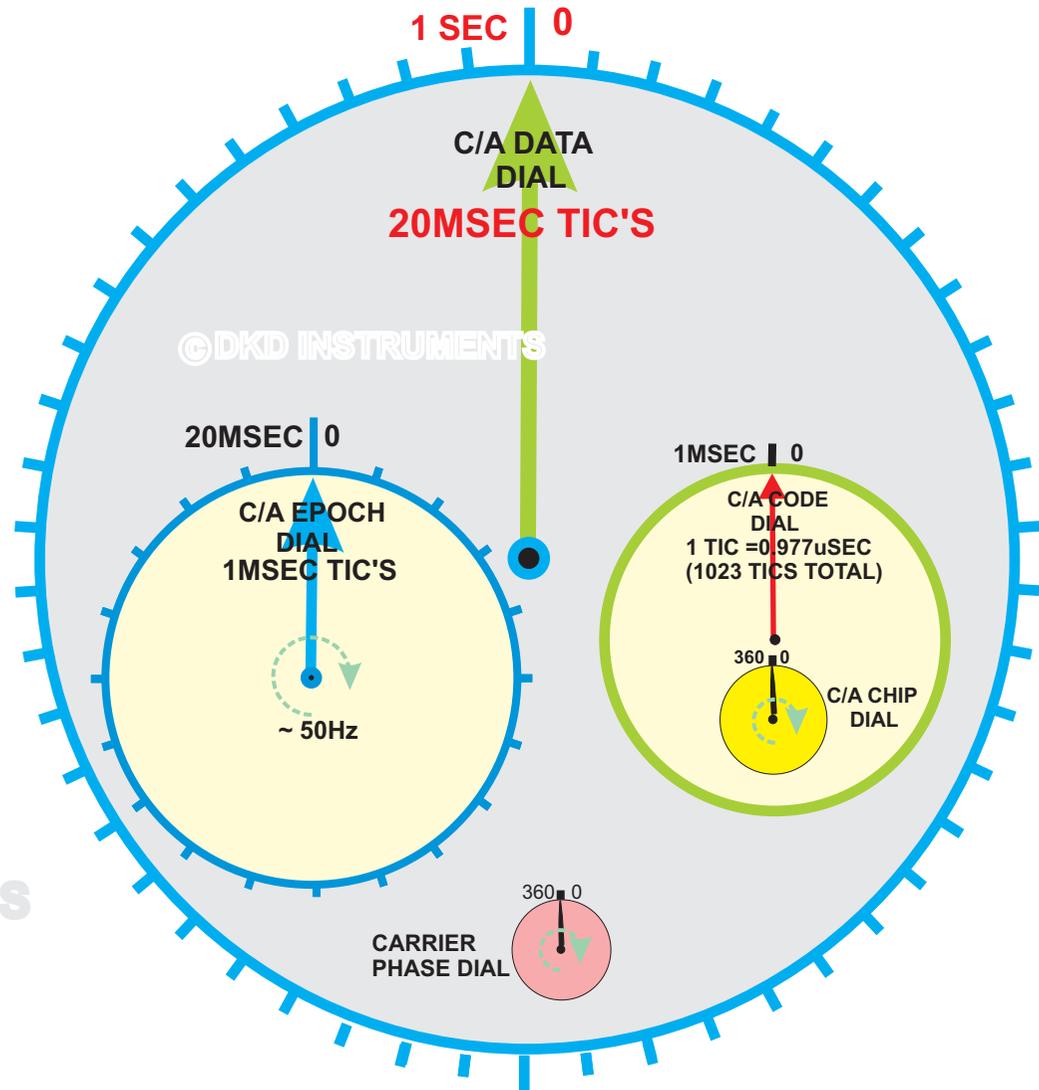


THE GPS L1 C/A CLOCK IS A COMMENSURATE CLOCK with ALL DIALS IN SIMPLE INTEGER RATIO'S

WITH ALL DIALS RELATED BY SIMPLE INTEGERS THE GPS CLOCK IS COMMENSURATE IN THAT ALL DIALS WILL READ ZERO @ 1SEC DIAL ZERO MARK.

THE RATIOS OF THE DIALS ARE;

- 50:1 C/A EPOCH DIAL to 1 SEC DIAL
- 20:1 C/A CODE DIAL to C/A EPOCH DIAL
- 1023:1 C/A CHIP DIAL to C/A CODE DIAL
- 1540:1 CARRIER PHASE to C/A CHIP DIAL



CLOCKS THAT SAMPLE OTHER CLOCKS SHOULD HAVE A NON-COMMENSURATE RATE RELATIONSHIP

AT THE STROBE INSTANT THE PHASE OF CLOCK2 IS CAPTURED FOR SUBSEQUENT USE BY SIGNAL PROCESSING. THIS PROCESS IS TYPICAL OF GPS RECEIVER INPUT SIGNAL SAMPLING OR DIGITIZING.

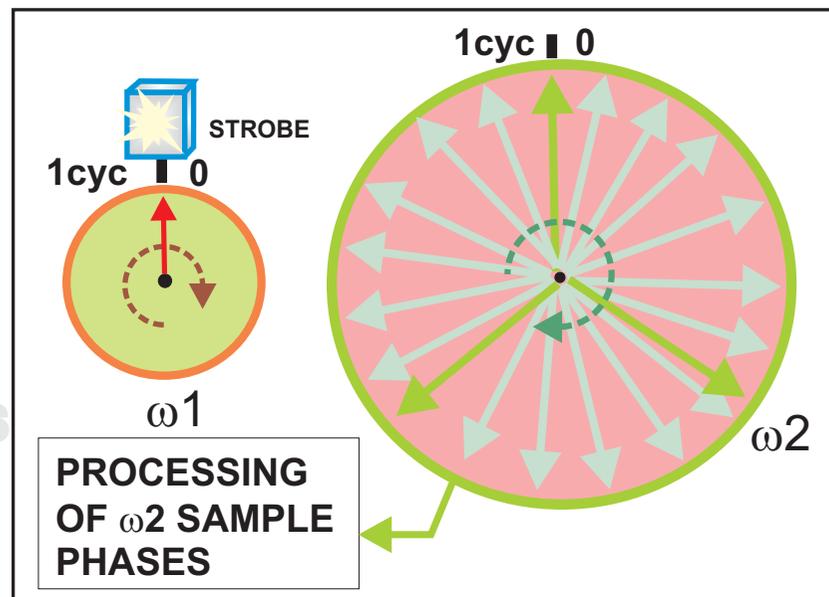
IF THE DIAL_RATIO IS COMMENSURATE ONLY A LIMITED NUMBER OF PHASE STATES OF CLOCK2 COULD BE SAMPLED. FOR EXAMPLE FOR $N=3$ ONLY 3 PHASE STATES OF CLOCK2 WOULD BE CAPTURED AS SHOWN AS DARK GREEN IN THE FIGURE BELOW. THE SHADED PHASE STATES OF CLOCK2 INDICATE A HIGHER LEVEL OF NON COMMENSURATE SAMPLING.

FOR PHASE SAMPLING OF ONE CLOCK BY ANOTHER A HIGH DEGREE OF NON COMMENSURATE SAMPLING IS USUALLY DESIRABLE.

THE NUMBER OF DIFFERENT PHASE SAMPLES WHEN THE DIAL_RATIO IS A RATIO OF TWO INTEGERS IS EQUAL TO N_{factor}

PHASE SAMPLING OF
CLOCK2 BY CLOCK1@
STROBE INSTANT

©DKD INSTRUMENTS

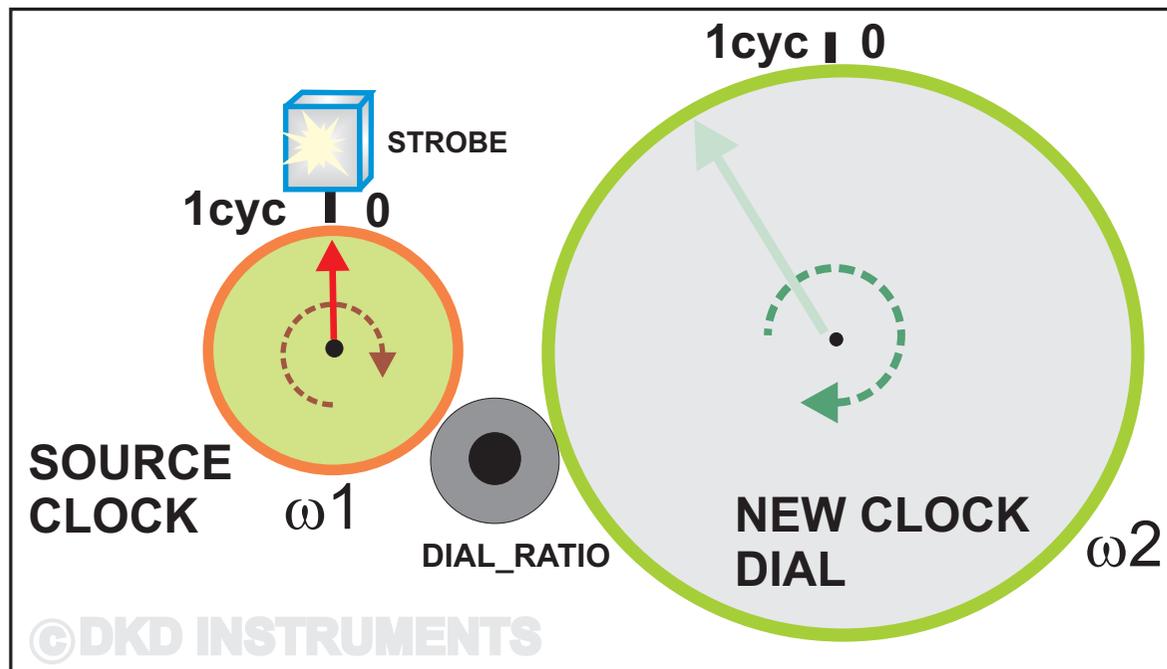


ALL DIGITAL TYPE CLOCKS CREATED FROM A SOURCE CLOCK DIAL (SPECIFICALLY NUMERICALLY CONTROLLED OSCILLATOR OR NCO) OFTEN HAVE A NON -COMMENSURATE RATE RELATIONSHIP

FOR THIS TYPE OF CLOCK GENERATION A NON COMMENSURATE RATE RELATIONSHIP IS OFTEN SELECTED FOR DIAL_RATIO.

WITH A HIGHLY NON COMMENSURATE DESIGN THE NEW CLOCK WILL HAVE MANY PHASES WITH RESPECT TO THE SOURCE CLOCK AS CAPTURED AT THE STROBE INSTANT.

FOR THE COMMENSURATE CASE ONLY A FEW PHASE RELATIONS WOULD EXIST. THIS CAN CREATE UNWANTED CLOCK RELATED SIGNALS IN TYPICAL GPS DSP METHODS.



MULTI-DIAL ROTARY CLOCKS SUMMARY

MULTI DIAL ROTARY CLOCKS CAN PERFORM INTEGRATION, DIFFERENTIATION, RATE DIVISION, RATE MULTIPLICATION, PHASE DIVISION and PHASE MULTIPLICATION.

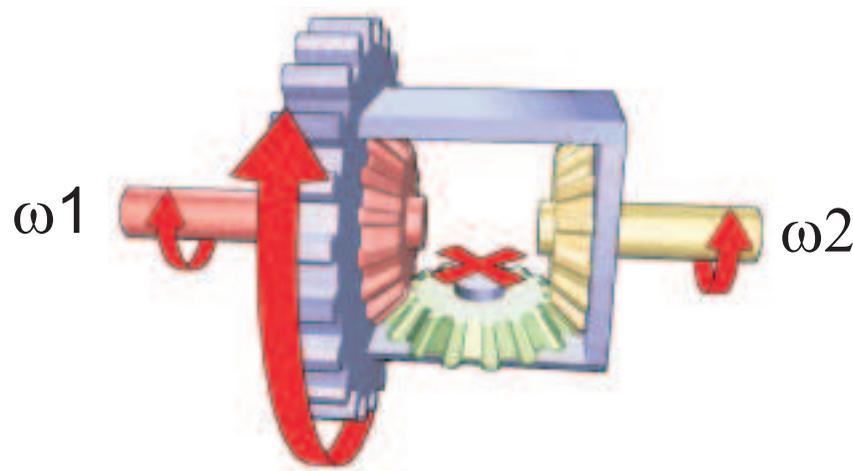
COMMENSURATE AND NON COMMENSURATE RATES ARE NEEDED IN GPS RECEIVERS. IN TYPICAL GPS SIGNAL PROCESSING THE ~NON COMMENSURATE RATE'S ARE EXECUTED WITH LARGE VALUES OF N_{factor}



APPENDIX III
DIFFERENTIAL GEAR MECHANISMS
CLOCK RATE AND PHASE DIFFERENCES
RATE AND PHASE LOCKED DIALS
CYCLE SLIPPING
&
QUADRATURE PROCESSING

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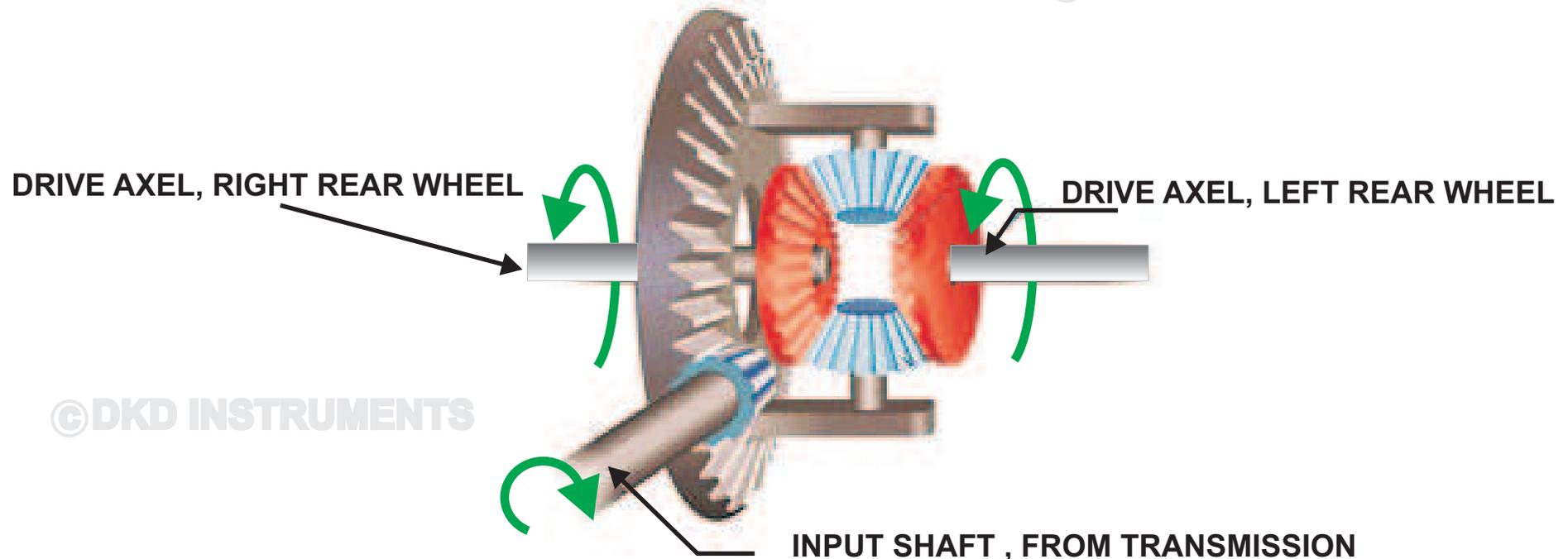
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THE DIFFERENTIAL GEAR MECHANISM FOUND IN MOST CARS AND OTHER FOUR WHEEL VEHICLES IS CAPABLE OF PERFORMING RATE AND PHASE DIFFERENCES.

IN AUTOS THE INPUT SHAFT IS TURNED BY THE ENGINE AND DRIVE AXLES ARE OUTPUTS TO THE REAR WHEELS , AS SHOWN. THE DIFFERENTIAL ALLOWS THE REAR WHEELS TO ROTATE AT DIFFERENT SPEEDS

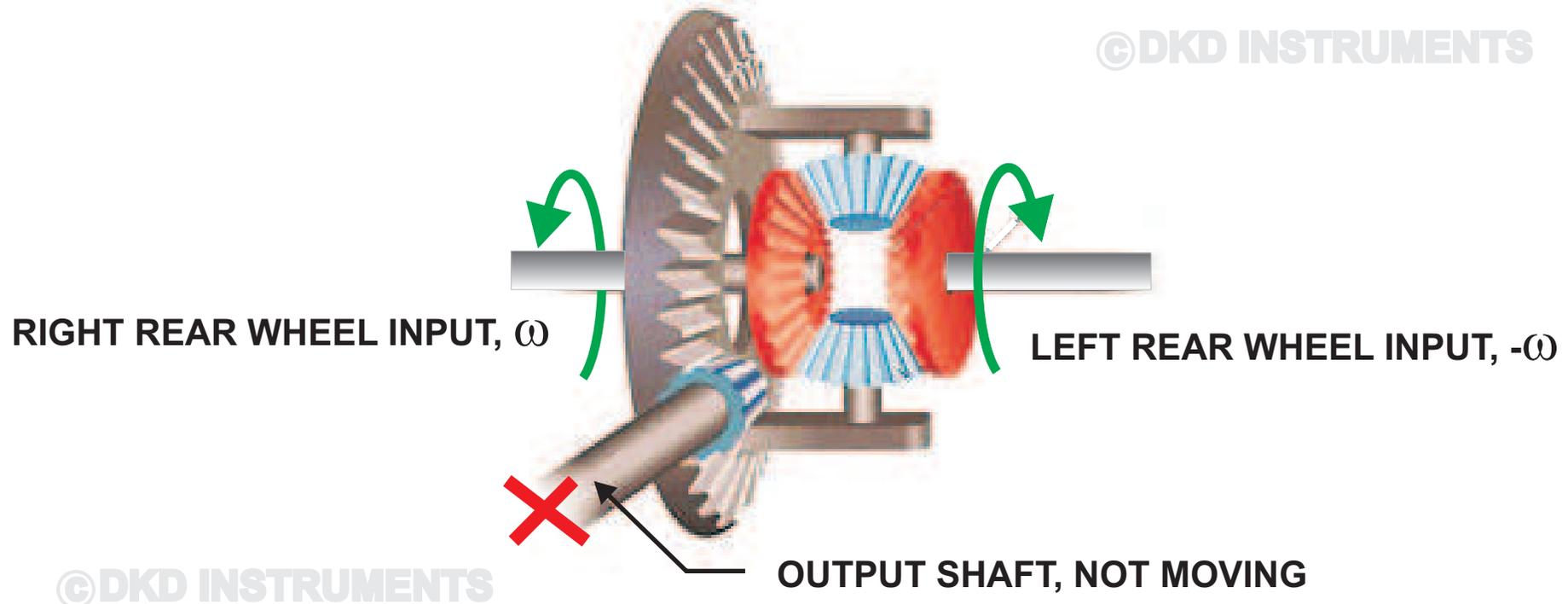
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WE NOW CONFIGURE THE DIFFERENTIAL WITH INPUTS ON THE AXEL SHAFTS. THE INPUT SHAFT IS NOW AN OUTPUT.

IF THE LEFT AND RIGHT WHEELS ARE ROTATED AT THE SAME RATE, ω , BUT IN OPPOSITE DIRECTIONS, INDICATED BY $-\omega$, THE OUTPUT SHAFT WILL NOT ROTATE.

AS CONFIGURED HERE THE DIFFERENTIAL IS PERFORMING RATE SUM OF THE TWO INPUT RATES, WHICH IN THIS CASE IS ZERO.



IF WE PROVIDE A REVERSING GEAR ON ONE OF THE INPUTS AND SET ALL RATIOS TO 1:1 WE HAVE A USEFUL TOOL FOR INVESTIGATING RATE AND PHASE RELATIONSHIPS BETWEEN TWO DIFFERENT CLOCKS OR BETWEEN TWO DIFFERENT CLOCK DIALS.

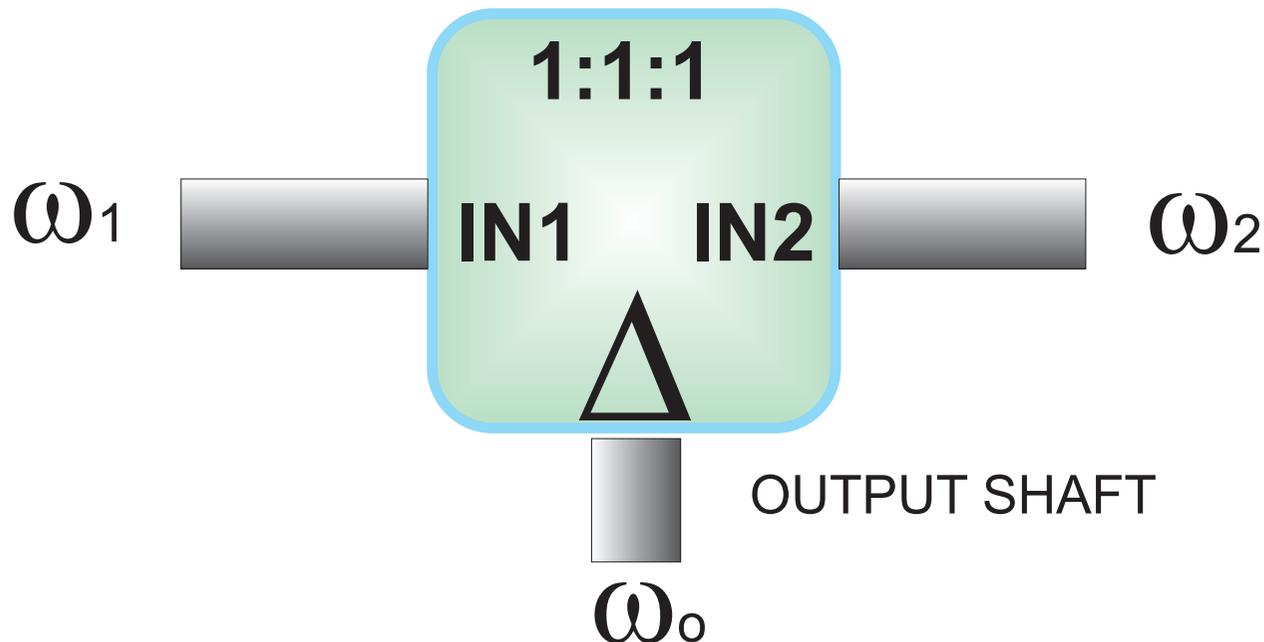
THE SCHEMATIC SYMBOL BELOW REPRESENTS A DIFFERENTIAL MECHANISM WITH TWO INPUTS, ω_1 & ω_2 AND ONE OUTPUT, ω_o . WE ARBITRARILY DECIDE TO HAVE CLOCKWISE ROTATION OF THE OUTPUT SHAFT IF $\omega_1 > \omega_2$.

THE OUTPUT ω_o , IS THE RATE AND PHASE DIFFERENCE OF THE INPUTS.

IF $\omega_1 = \omega_2$ THEN $\omega_o = 0$.

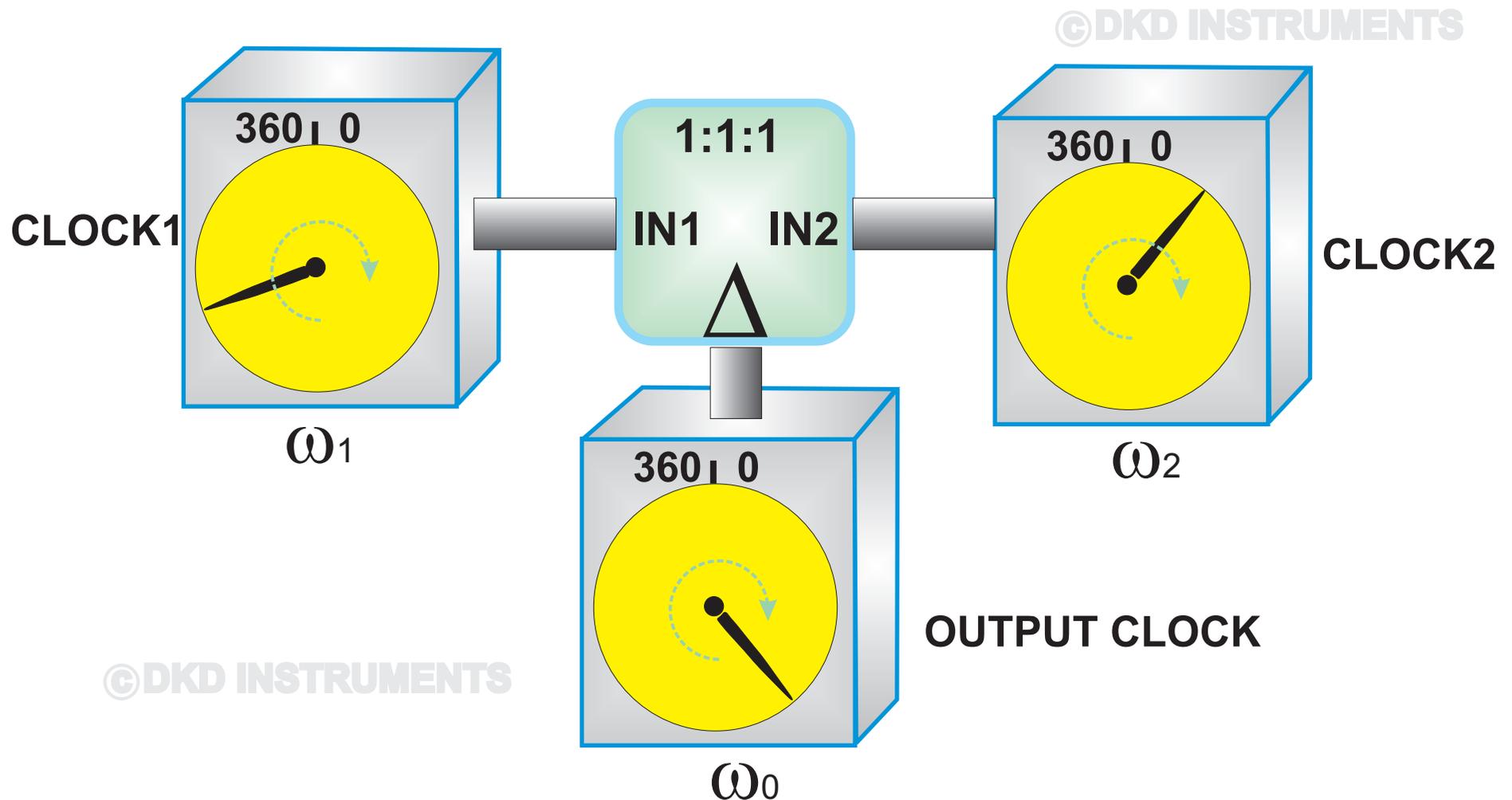
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IF TWO CLOCKS, WITH RATES , ω_1 & ω_2 , , ARE ATTACHED TO THE INPUT SHAFTS OF THE DIFFERENTIAL WE CAN CREATE A NEW CLOCK THAT HAS A NEW RATE OF ω_0 .

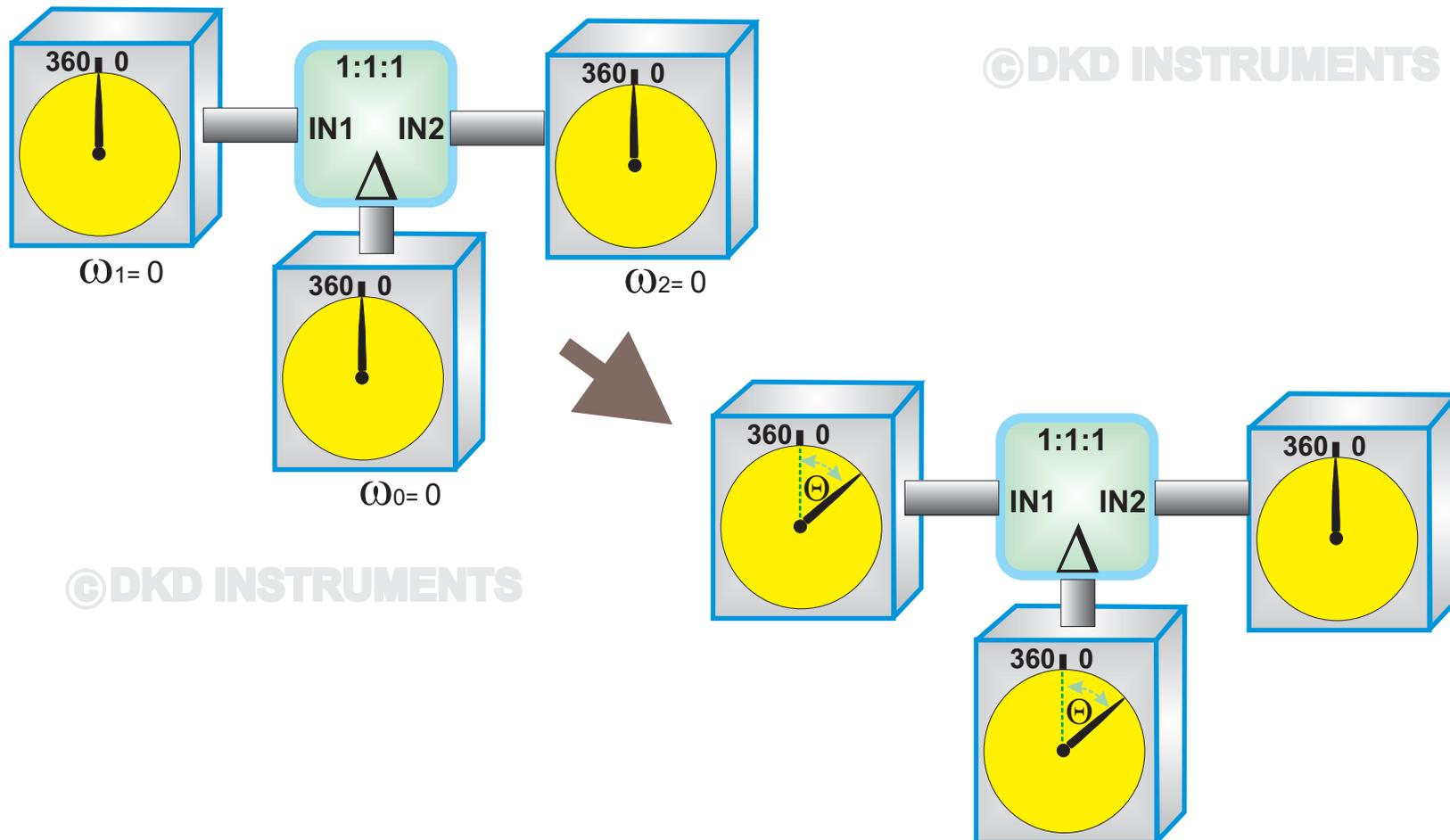
$\omega_0 = \omega_1 - \omega_2$; AS SHOWN $\omega_1 > \omega_2$, ω_0 IS ROTATING CLOCK WISE



PHASE OFFSET IS PRESERVED(STATIC DIFFERENTIAL)

NOW WE STOP THE TWO INPUT CLOCKS FROM ROTATING, ω_1 , ω_2 , $\omega_0 = 0$, AND SET ALL DIALS TO ZERO PHASE. IF WE NOW ROTATE CLOCK1 BY AN ANGLE Θ THE OUTPUT CLOCK DIAL ROTATES BY Θ .

THE DIFFERENTIAL COMPUTES THE PHASE DIFFERENCE BETWEEN CLOCK 1 AND CLOCK 2 AND DISPLAYS THE RESULT ON THE OUTPUT CLOCK. THIS OCCURS EVEN IF THE CLOCKS ARE NOT ROTATING.



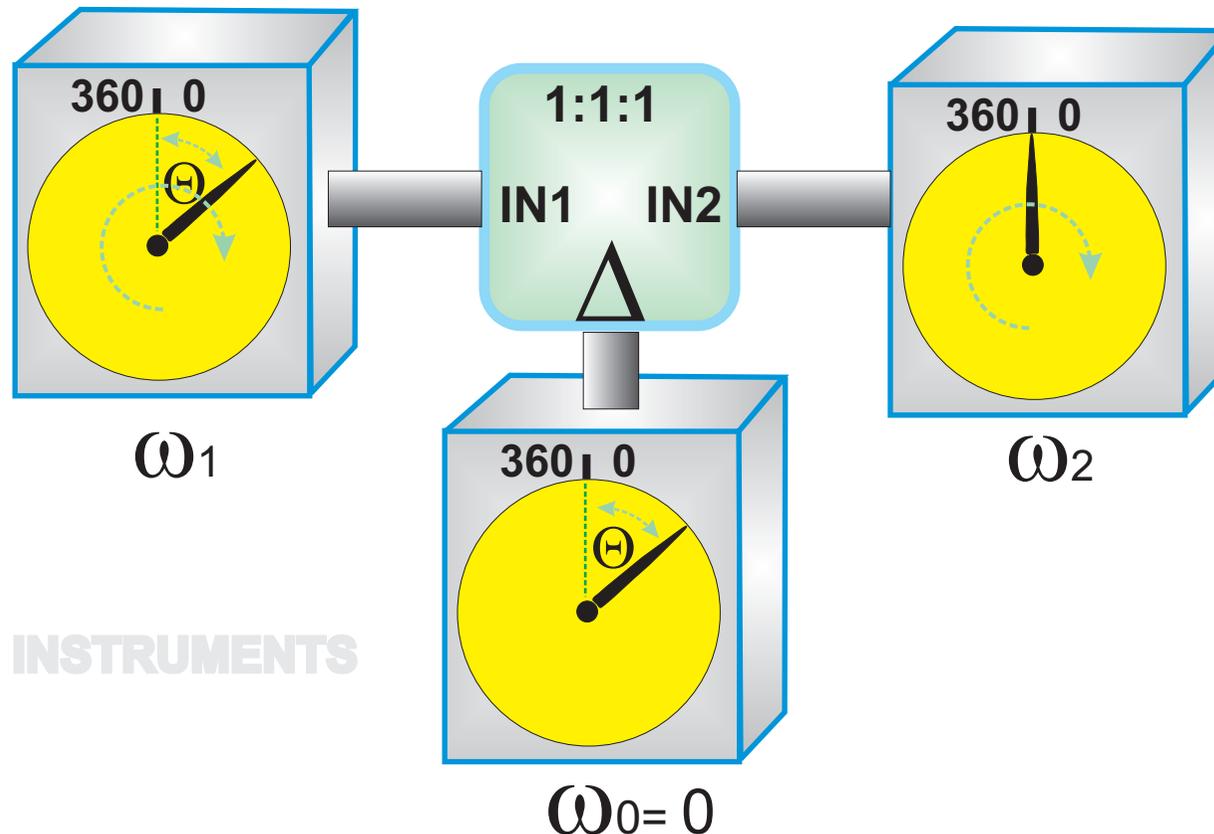
PHASE OFFSET PRESERVED WHEN ROTATING

NOW WE START CLOCK 1 AND 2 ROTATING AT THE SAME RATE $\omega_1 = \omega_2$. WITH THE TWO INPUT RATES BEING EQUAL THE OUTPUT RATE $\omega_0 = 0$.

EVEN THOUGH THE INPUTS ARE NOW ROTATING THE ORIGINAL PHASE OFFSET Θ IS PRESERVED IN CLOCK 1 RELATIVE TO CLOCK 2.

ALSO THE OUTPUT CLOCK DIAL IS STILL OFFSET BY Θ AND IS NOT MOVING SINCE INPUT RATES ARE THE SAME.

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RATE LOCKED LOOP

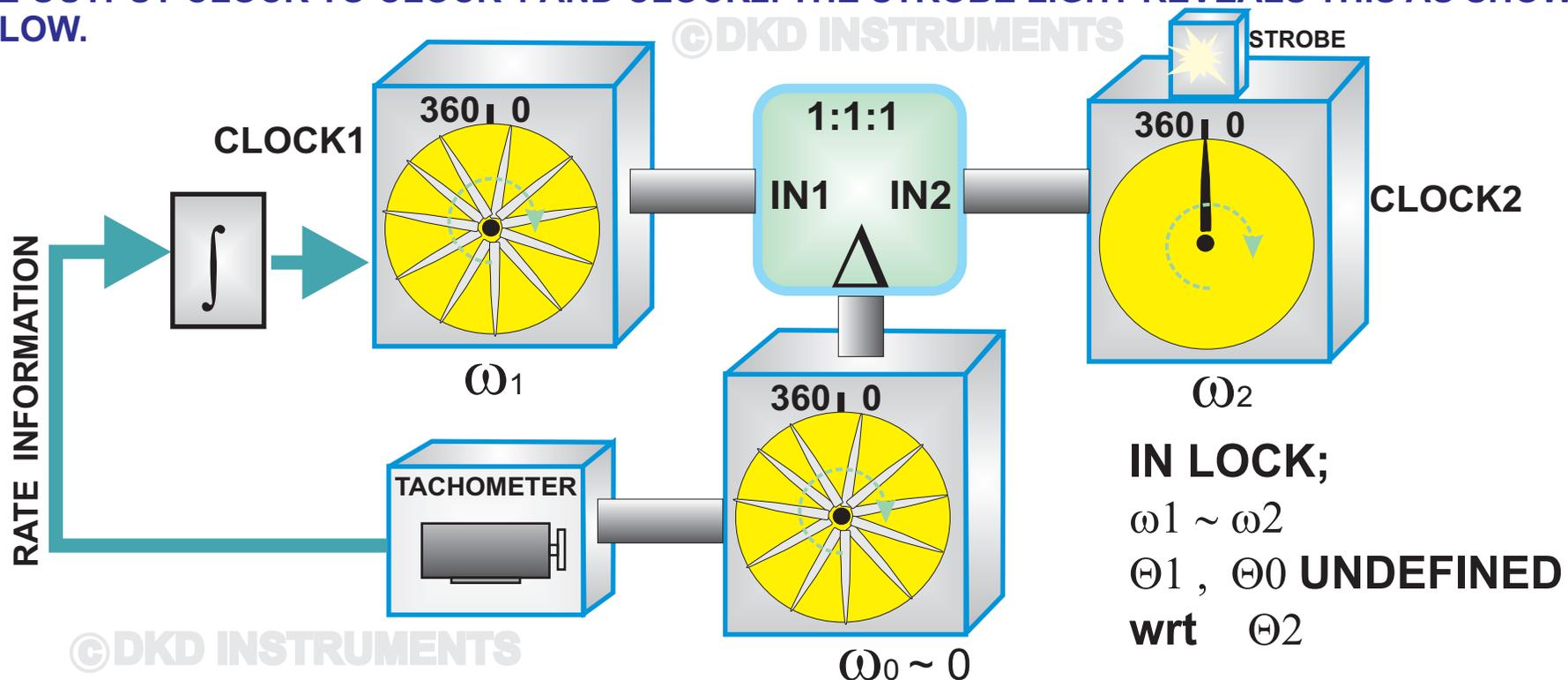
WE WISH TO MAKE THE RATE OF CLOCK 1 EQUAL TO CLOCK 2, ie IN SYNTONY.

WE ATTACH A TACHOMETER TO THE OUTPUT CLOCK. THE TACHOMETER PRODUCES A POSITIVE VOLTAGE WHEN CLOCK 1 RATE IS LARGER THAN CLOCK 2 RATE AND A NEGATIVE VOLTAGE WHEN CLOCK 1 RATE IS LESS THAN CLOCK 2 RATE. WHEN RATES ARE ~EQUAL VOLTAGE IS ~ ZERO. THIS VOLTAGE IS INTEGRATED AND APPLIED TO THE RATE CONTROL ON CLOCK 1 .

A STROBE LIGHT IS CONNECTED CLOCK 2 SUCH THAT EACH TIME IT'S POINTER CROSSES ZERO THE STROBE IS TRIGGERED SO AS TO CAPTURE THE PHASE RELATIONSHIPS..

WHEN CLOCK1 IS RATE LOCKED ITS RATE IS EQUAL TO CLOCK2 RATE(IN AN AVERAGE SENSE). THE AVERAGE RATE OF THE OUTPUT CLOCK IS THEREFORE ZERO.

THE PHASE RELATION OF CLOCK 1 TO CLOCK 2 IS UNDEFINED AS IS THE PHASE RELATIONSHIP OF THE OUTPUT CLOCK TO CLOCK 1 AND CLOCK2. THE STROBE LIGHT REVEALS THIS AS SHOWN BELOW.



RATE STEERED PHASE LOCKED LOOP (2π TYPE PHASE DETECTOR)

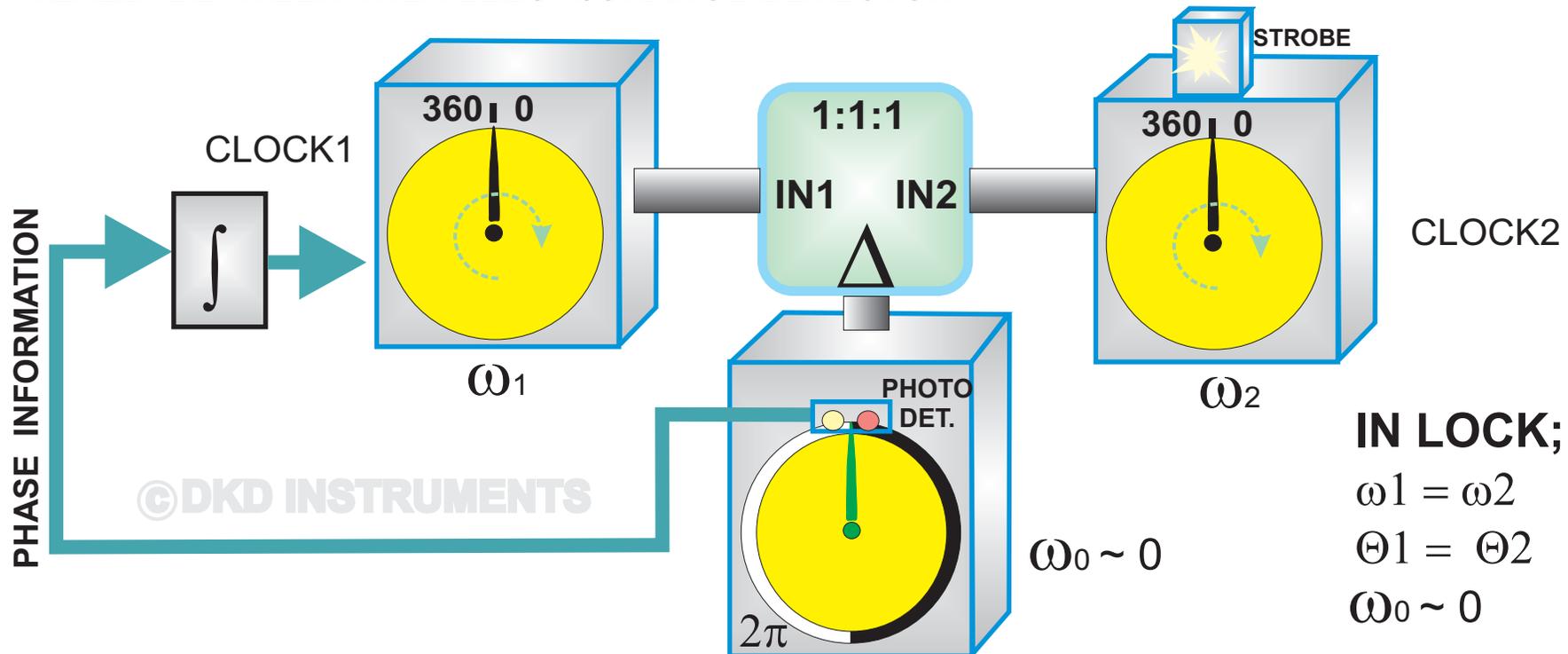
WE NOW WISH TO CONTROL CLOCK 1 RATE SUCH THAT THE OUTPUT CLOCK DIAL STOPS MOVING AND REMAINS POINTING AT ZERO. WHEN THIS STATE IS MAINTAINED CLOCK1 IS PHASE LOCKED TO CLOCK2. THE PHASE STATES BELOW REPRESENT WHAT THE STROBE WOULD REVEAL.

TO ACCOMPLISH THIS WE MODIFY THE OUTPUT CLOCK WITH A BLACK/WHITE RIM AND A PHOTO SENSOR. THE BLACK/WHITE RIM ROTATES WITH THE POINTER.

THE PHOTO SENSOR HAS TWO DETECTORS VERY CLOSE TO EACH OTHER. PHASE STEADY STATE IS ACHIEVED ONLY WHEN THE YELLOW DETECTOR IS OVER THE WHITE SECTION OF THE RIM AND . WHEN THE ORANGE DETECTOR IS OVER THE BLACK PART OF THE RIM. THIS ONLY OCCURS AT ONE PLACE ON THE RIM, HENCE IT IS A 2π DETECTOR.

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TO KEEP THE PHASE OF CLOCK1 EQUAL TO PHASE OF CLOCK2 THE LOOP CONTROLS THE RATE OF CLOCK1 , UP OR DOWN AS NEEDED, TO KEEP THE WHITE/BLACK TRANSITION OF THE RIM CENTERED BETWEEN THE YELLOW/ORANGE DETECTOR

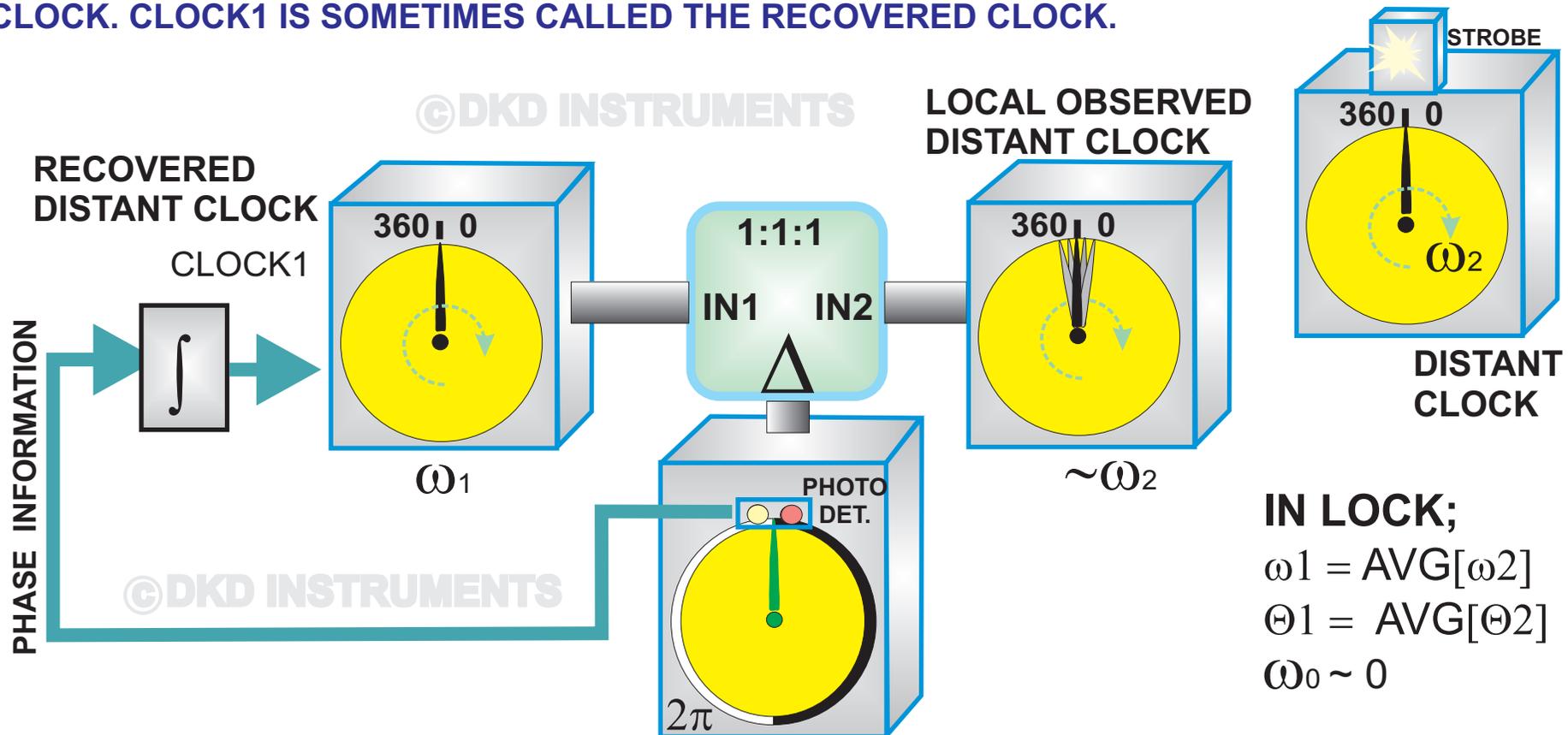


PHASE LOCK LOOP WITH NOISY REFERENCE

OFTEN THE CLOCK WE ARE LOCKING TO IS AN OBSERVED CLOCK FROM A DISTANT TRANSMITTER. SUCH IS THE CASE WITH GPS. AT THE TRANSMITTER THE PHASE OF THE CLOCK IS FREE OF DISTURBANCE. THE TRANSMISSION AND RECEPTION PROCESS OF A DISTANCE CLOCK SIGNAL INTRODUCE ERRORS IN PHASE.

A PHASE LOCK LOOP CAN BE USED TO CLEAN UP A NOISY RECEIVED CLOCK. BELOW WE SEE THE DISTANT REFERENCE CLOCK WITH THE STROBE MOUNTED ON IT. AS WE CAN SEE THE LOCAL OBSERVED CLOCK2 (OUR LOCAL OBSERVED REFERENCE CLOCK) HAS PHASE NOISE ON IT WRT TO ORIGINAL TRANSMITTED CLOCK.

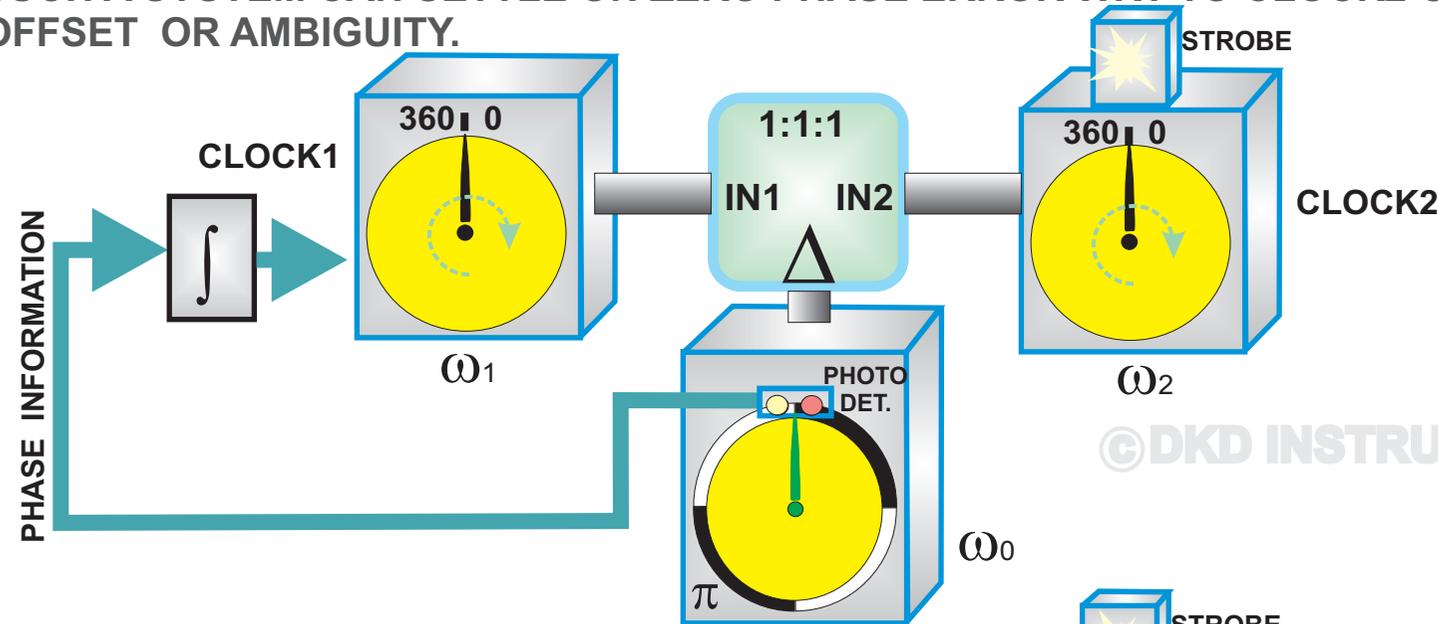
THE PHASE LOCKED LOOP ACTS LIKE A FILTER ON THE OBSERVED CLOCK PHASE. THE RESULT IS THAT CLOCK1 HAS A BETTER ESTIMATE OF THE TRUE PHASE OF THE DISTANT REFERENCE CLOCK. CLOCK1 IS SOMETIMES CALLED THE RECOVERED CLOCK.



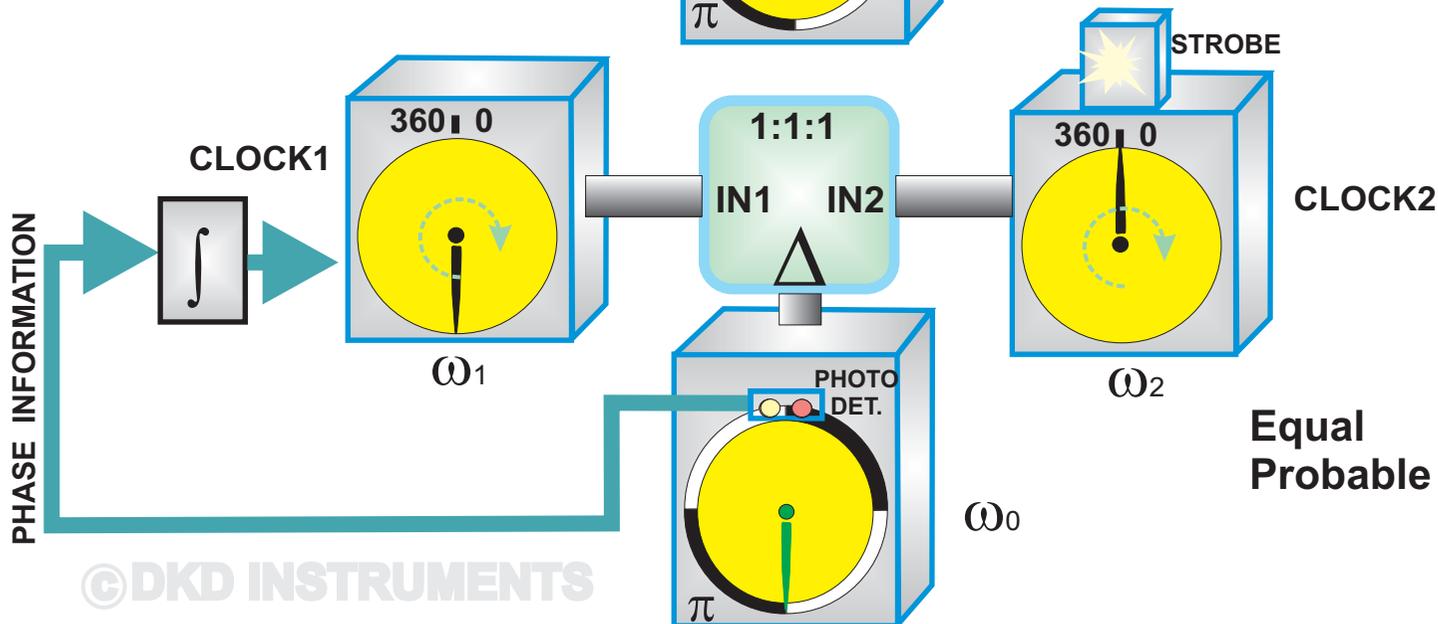
RATE STEERED PHASE LOCKED LOOP WITH 180 DEGREE AMBIGUITY (π PHASE TYPE DETECTOR)

IF WE REPLACE THE HALF CIRCLE BLACK/WHITE RIM ON THE OUTPUT CLOCK DIAL WITH ONE THAT IS DIVIDED IN QUARTERS WE CREATE A PHASE LOCK SYSTEM THAT HAS TWO STABLE OPERATING POINTS WHEN IN LOCK.

SUCH A SYSTEM CAN SETTLE ON ZERO PHASE ERROR WRT TO CLOCK2 OR WITH 180 DEGREES OFFSET OR AMBIGUITY.



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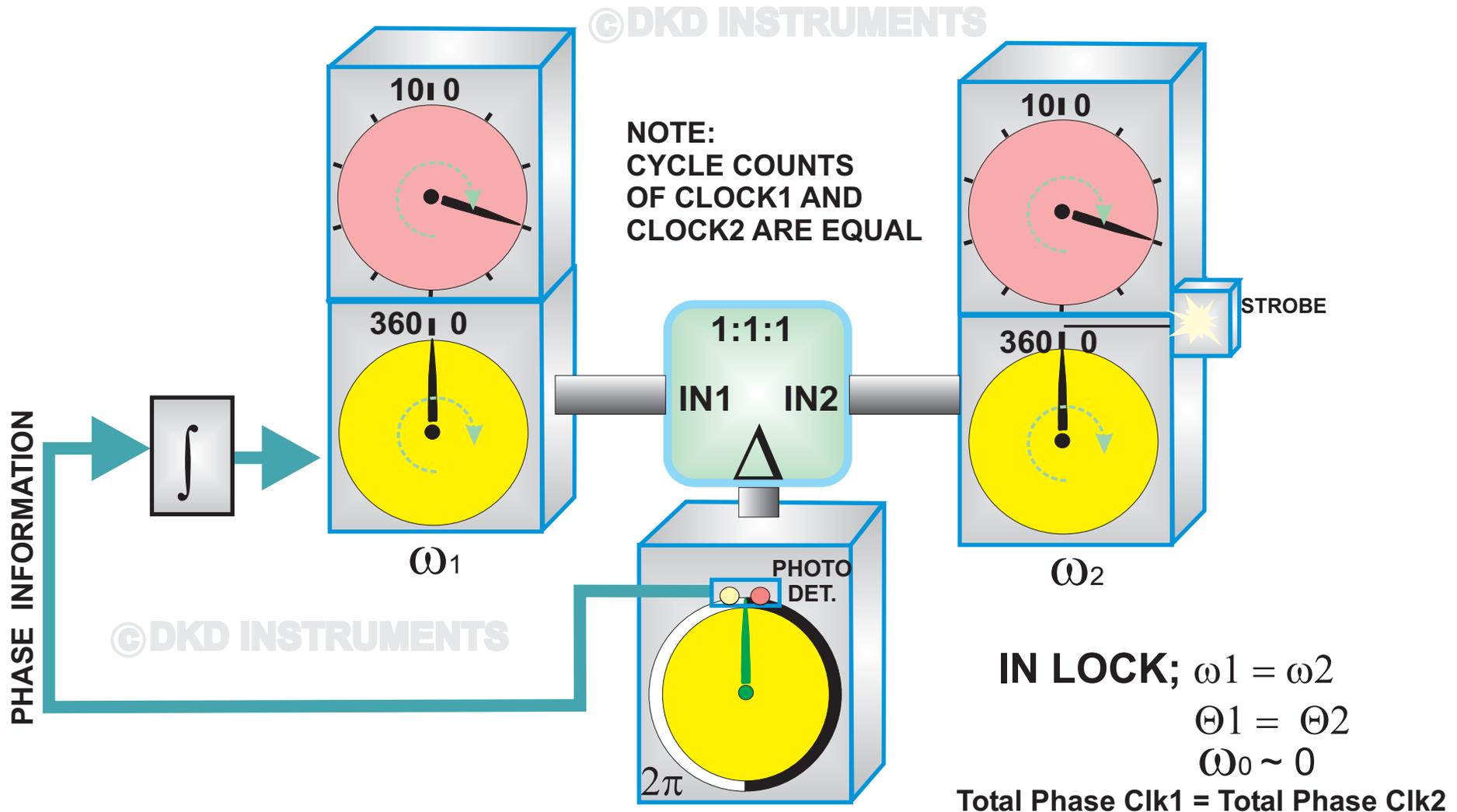
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IN LOCK;
 $\omega_1 = \omega_2$
Equal Probable $\left\{ \begin{array}{l} \theta_1 = \theta_2 \\ \theta_1 \text{ Or } \theta_2 + 180\text{Deg} \end{array} \right.$
 $\omega_0 \sim 0$

CYCLE SLIPPING IN PLL

OUR PHASE LOCKED CLOCKED SYSTEM CAN BE DISTURBED ENOUGH TO CAUSE THE LOCK TO BREAK. TO INVESTIGATE WHAT WILL HAPPEN TO OUR PLL WE ADD CYCLE COUNTERS TO CLOCK 1 AND 2, THESE COUNTERS COUNT UP TO 10 FULL CYCLES OF THEIR RESPECTIVE 0/360 DIALS.

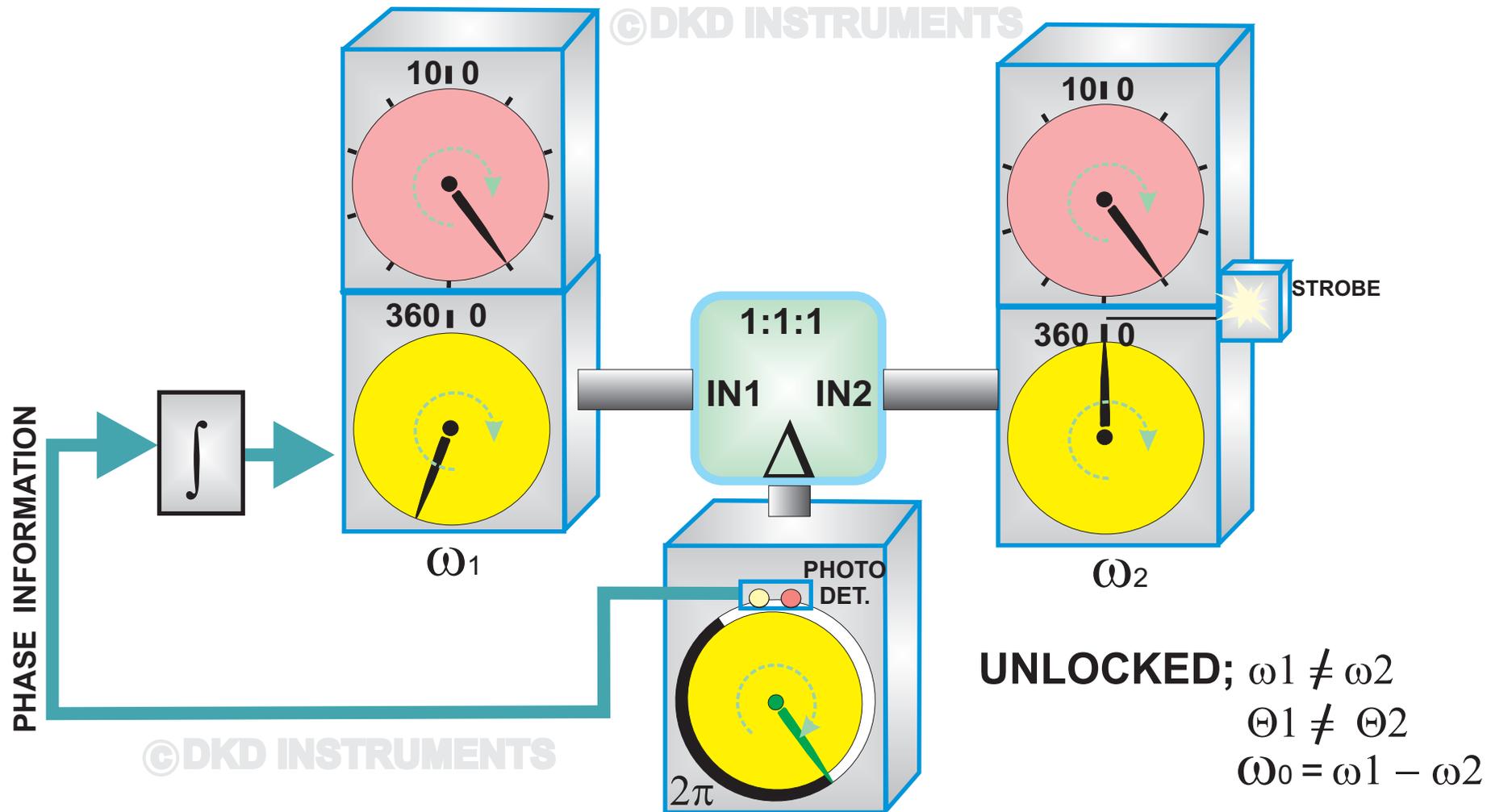
AS REVEALED BY THE STROBE ON CLOCK 2 THE SYSTEM BELOW IS IN LOCK WITH 3 FULL CYCLES ON BOTH CLOCK 1 AND CLOCK 2.



OUR PLL HAS BECOME UNLOCKED!

SOMETIME AFTER THE 4TH CYCLE OF CLOCK2 THE LOOP BECAME UNLOCKED. NOW THE OUTPUT CLOCK DIAL IS SPINNING AND THE RATE OF CLOCK1 IS NO LONGER EQUAL TO THE RATE OF CLOCK2.

TYPICALLY THE RATE OF CLOCK1 IN THE UNLOCK STATE CAN BE EITHER GREATER OR LESS THAN CLOCK2 RATE.

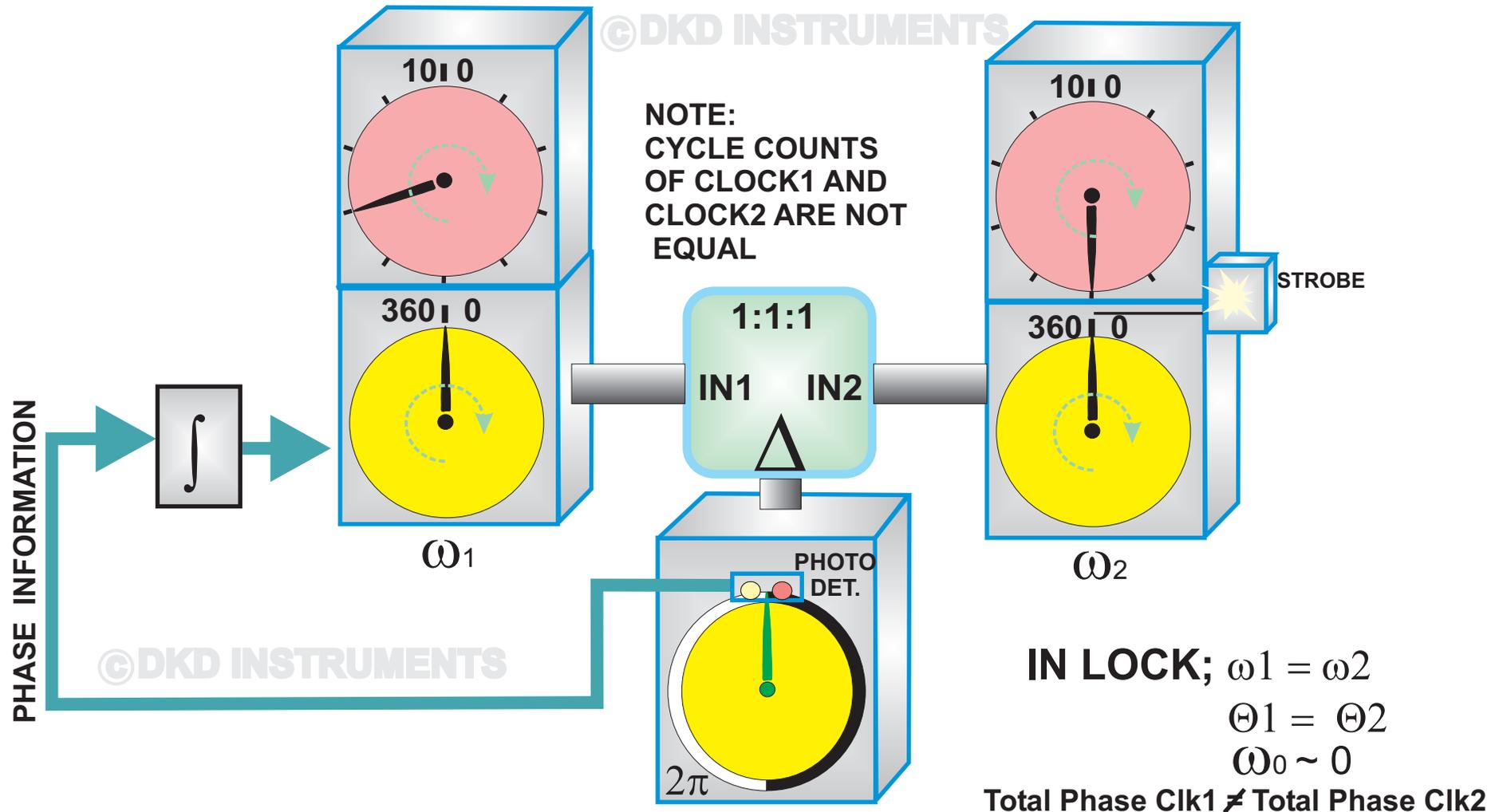


OUR PLL HAS REGAINED LOCK, BUT CYCLE SLIPS HAVE OCCURRED

OUR LOOP HAS REGAINED LOCK BUT IN THE PROCESS OF REGAINING LOCK THE UNCONTROLLED RATE OF CLOCK1 ALLOWED IT TO ADVANCE PAST CLOCK2 BY 2 FULL CYCLES.

THIS IS ONE TYPE OF WHAT IS CALLED **CYCLE SLIPPING**

NOTE THAT EVEN THOUGH THE PHASE OF ω_1 DIAL AND ω_2 ARE AGAIN EQUAL THE OVER ALL TIME OF CLOCK1 IN RELATION TO CLOCK2 IS NOT EQUAL. CLOCK1 IS AHEAD OF CLOCK2 BY TWO FULL CYCLES OF ITS 0/360 DIAL

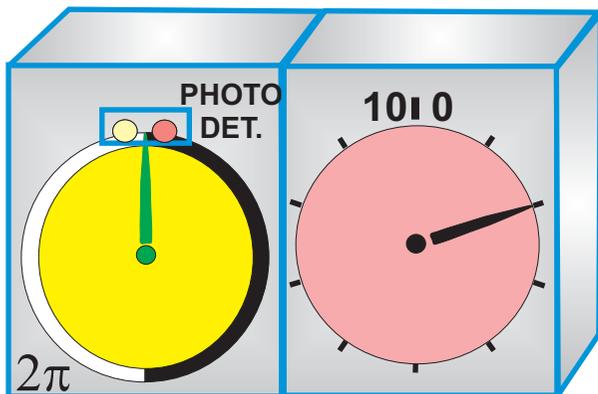


COMMENTS ON CYCLE SLIPS IN A PLL

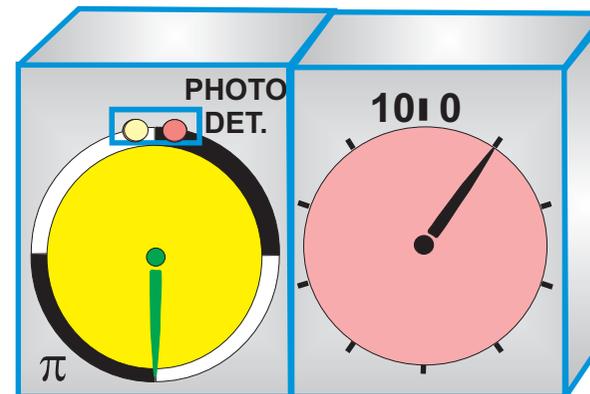
- ▶ IF WE INSTALLED A CYCLE COUNTER (THAT WAS SET TO ZERO BEFORE THE UNLOCK EVENT) ON THE OUTPUT CLOCK DIAL IT WOULD HAVE SHOWN TWO FULL CYCLES OF CYCLE SLIP RESULTING FROM THE UNLOCK EVENT , SEE FIGURE BELOW.
- ▶ IN GENERAL AFTER A PLL UNLOCK EVENT AND CYCLE SLIPS HAVE OCCURRED THE NEW STEADY STATE PHASE MAY BE IN ERROR BY +/- N FULL CYCLES, ASSUMING A 2π PHASE DETECTOR IS USED. [N= 1, 2,....]
- ▶ IF THE 2π TYPE PHASE DETECTOR DIAL WAS REPLACED WITH A π TYPE DETECTOR THEN THE NEW STEADY STATE PHASE MAY BE IN ERROR BY +/- N*[1/2 CYCLES] AFTER AN UNLOCK EVENT. [N= 1,2...], SEE SECOND FIGURE BELOW.
- ▶ CYCLE SLIPS ARE THE BANE OF GPS CARRIER PHASE MEASUREMENTS. IN GPS RECEIVERS MOST CARRIER PHASE DIAL TRACKING IS DONE BY COSTAS LOOP METHODS. COSTAS LOOP ARE OF THE π PHASE DETECTOR TYPE. THEREFORE IF THE COSTAS CARRIER LOOP BREAKS LOCK EXPECT +/-N*[1/2 CYCLE] PHASE JUMPS IN THE CARRIER PHASE DATA.

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2 CYCLE SLIPS INDICATED



**1 AND 1/2 CYCLE SLIPS INDICATED
(ASSUMING INITIAL LOCK WAS AT ZERO DEGREES)**



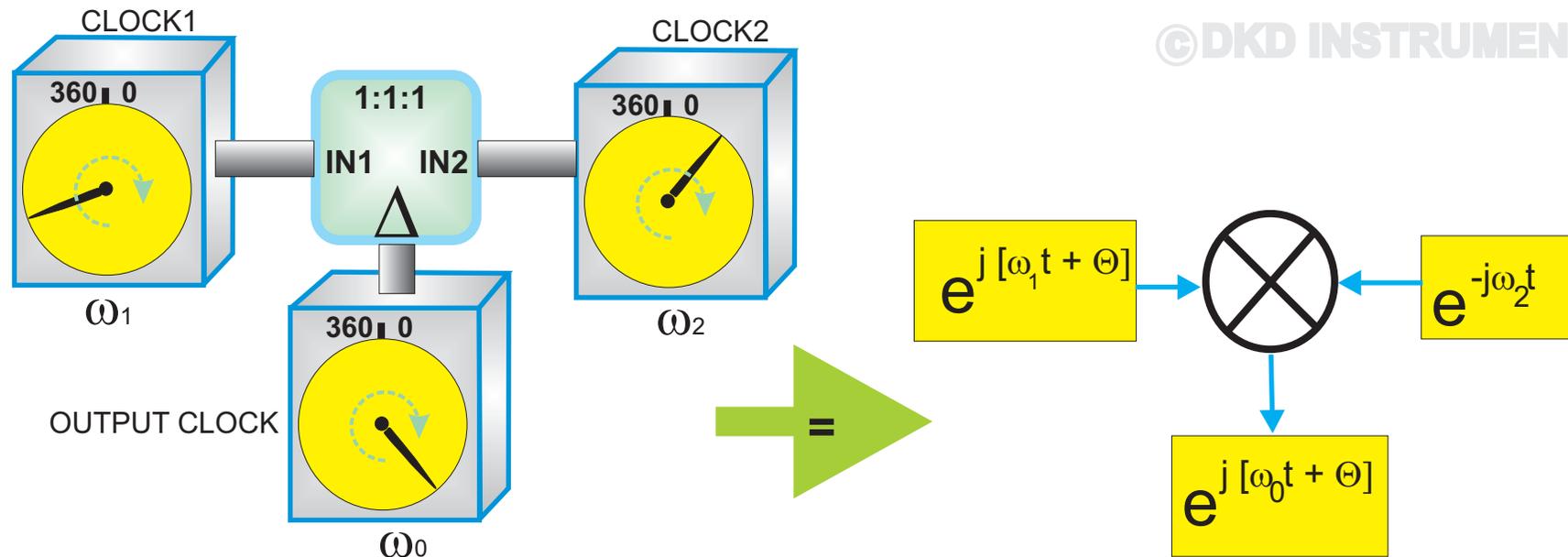
MATH BEHIND THE DIFFERENTIAL GEAR MECHANISM

THE CLOCK INPUTS TO THE DIFFERENTIAL GEAR CAN BE MODELED AS PHASOR INPUTS. THE OUTPUT OF CLOCK OF THE DIFFERENTIAL IS ALSO A PHASOR WITH ITS RATE OF ROTATION EQUAL TO THE DIFFERENCE OF THE INPUT RATES.

THIS CAN ONLY OCCUR IF ONE INPUT PHASOR IS REVERSED IN ROTATION (NEGATIVE SIGN) AND THE TWO INPUT PHASORS MULTIPLIED. THE DIFFERENTIAL IS MULTIPLYING TWO CLOCKS AND PRODUCING A NEW CLOCK.

THE NEGATIVE SIGN ON ω_2 RESULTS FROM THE REVERSING GEAR IN THE DIFFERENTIAL SYMBOL ON IN2 PORT.

THE ANGLE Θ IS THE INITIAL ANGLE OFFSET DISCUSSED PREVIOUSLY. IF ω_1 AND ω_2 EQUAL ZERO WE SEE OUTPUT IS $[e^{j\Theta}]$. THIS IS THE SAME RESULT OBSERVED WITH THE STATIC DIFFERENTIAL.



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where; $\omega_0 = \omega_1 - \omega_2$
and Θ is initial angle of
Clock1 w.r.t Clock2

WHY QUADRATURE PROCESSING?

QUADRATURE PROCESSING IS OFTEN A SOURCE OF CONFUSION. IT IS OFTEN ASSUMED THE READER UNDERSTANDS WHAT THIS TERM MEANS AND WHY ITS BEING DONE.

THE REASON FOR QUADRATURE PROCESSING IS THIS:

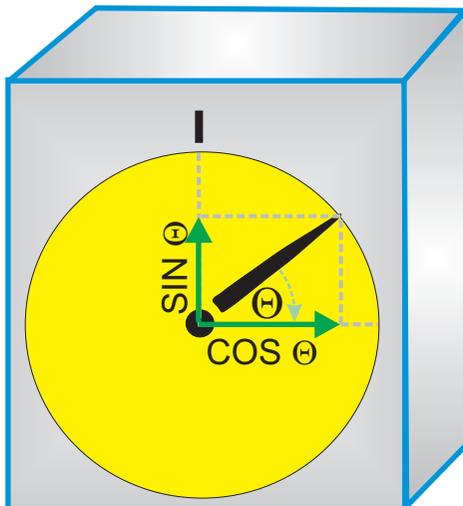
QUADRATURE PROCESSING IS DONE IN ORDER TO RECOVER THE *ANGLE OR PHASE* INFORMATION OF A CLOCK DIAL OR PHASOR.

IT IS POSSIBLE TO DETERMINE THE *RATE* OF A CLOCK DIAL (OR PHASOR) WITHOUT QUADRATURE PROCESSING, BUT NOT *THE DIRECTION OF ROTATION, OFFSET ANGLE/PHASE FROM A GIVEN PHASE REFERENCE OR DEMODULATE PHASE MODULATION PRESENT (e.g GPS BPSK DATA MOD)*.

IT TAKES *TWO* VECTORS IN *QUADRATURE* TO DESCRIBE A CLOCK DIAL OR PHASOR. IF WE USE JUST THE HORIZONTAL VECTOR OR JUST THE VERTICAL VECTOR THE PHASE INFORMATION IS LOST. IN THE ROTATING CASE EITHER HORIZONTAL OR VERTICAL VECTOR CONTAINS THE RATE INFORMATION.

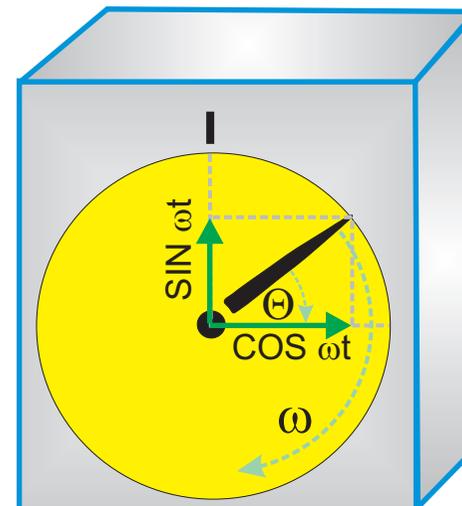
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STATIC CLOCK DIAL



$$\theta = \text{ARCTAN} [\text{SIN}\theta/\text{COS}\theta]$$

**CLOCK DIAL ROTATING
CLOCKWISE AT RATE ω**



$$\theta(t) = \text{ARCTAN} [\text{SIN}\omega t/\text{COS}\omega t]$$

SUMMARY OF DIFFERENTIAL GEAR MECHANISMS

CLOCK RATE AND PHASE DIFFERENCES

RATE AND PHASE LOCKED DIALS

CYCLE SLIPPING & QUADRATURE PROCESSING

- ▶ DIFFERENTIAL GEARS CAN BE USED TO PRODUCE A NEW CLOCK THAT DISPLAYS THE RATE AND PHASE DIFFERENCES OF THE TWO INPUT CLOCKS
- ▶ THE OUTPUT SHAFT DOES NOT ROTATE IF INPUT RATES ARE EQUAL AND SAME DIRECTION FOR THE SCHEMATIC DIFFERENTIAL
- ▶ INPUT PHASE OFFSETS ARE PRESERVED IN BOTH STATIC AND ROTATING OUTPUTS OF THE DIFFERENTIAL
- ▶ THE DIFFERENTIAL OUTPUT CLOCK RATE CAN BE MEASURED AND USED TO PRODUCE A RATE LOCKED LOOP.
- ▶ IF A PHASE DETECTOR IS ADDED TO OUTPUT CLOCK THE DIFFERENTIAL SYSTEM CAN BE CONFIGURED AS A PHASE LOCK SYSTEM OF ONE INPUT CLOCK TO THE OTHER INPUT CLOCK
- ▶ A PHASE LOCK LOOP ACTS AS A SMOOTHING FILTER ON A NOISY REFERENCE CLOCK PRODUCING A LOCKED CLOCK WITH LOWER PHASE DISTURBANCE.
- ▶ IF THE 2π PHASE DETECTOR IS REPLACED BY A π TYPE DETECTOR THERE ARE TWO EQUALLY LIKELY STABLE LOCK POINTS
- ▶ CYCLE SLIPS CAN OCCUR WHEN A PLL BREAKS LOCK AND RATE ERROR IS NON-ZERO ON THE CLOCK BEING LOCKED.
- ▶ CYCLE SLIPS CAN BE AN INTEGER MULTIPLE OF THE PHASE DETECTOR TYPE, FULL CYCLES, 1/2 CYCLES FOR 2π AND π DETECTORS RESPECTIVELY.
- ▶ THE INPUT CLOCKS CAN BE MODELED AS PHASORS AND THE DIFFERENTIAL GEAR IS ACTING AS A MULTIPLIER PRODUCING A NEW PHASOR, THE OUTPUT CLOCK.
- ▶ QUADRATURE PROCESSING ALLOWS PHASE INFORMATION TO BE RECOVERED. WHEN YOU SEE QUADRATURE SIGNAL PROCESSING THINK "PHASE INFORMATION RECOVERY"